UNITED SQUARE®

Educational Book 2
PUZZLES

Educational Book have been developed in collaboration with Stephane Robert, french Mathematic teacher.

UNITED SQUARE®

We now have six different available pieces, possibly in several copies, and we naturally began to observe them, to manipulate them, to or even turn them together.
Instinctively, our desire is strong with wanting to create beautiful colorful patterns and encountering triangles of the same color to form the multicolored image of square shimmering Harlequin's costume.

On geometric assembly, squares or rectangles, our imagination leads us to more graphic patterns. Some need regularity , in turn, leads us to look for more regular distribution of the colors or patterns more symmetrical. New challenges and questions or the search for a form of beauty is a pretext for handling work in preparing our future parties
United Square...

From now on, no more random and haphazard juxtapositions of parts against each other: two triangles may be contacted a long one side so that they are of the same color! The isosceles triangles are no longer isolated but combined to form squares. It therefore applies the strict game rule of the placement United Square pieces. A basic way to discover this main rule, but also and above all to prepare ahead of time without the strategic considerations of the best choices to make. The goal is simply to train to be able to answer the question: « At this location, what piece am I going to play? »

To start, especially with young children, it just has to offer puzzles to complete. Incomplete configuration is given, the unfinished figure by paving parts of the game. It should finish by placing in open spaces, the pieces, while respecting the necessary conditions for color matching. First, some parts to select from a limited set, it then moves to more choices and to cover larger areas.

The sheet « **Costume** » offers a first input relatively easy on this type of task. Just make them match colors that two single triangles with no double correspondence . Great simplicity and therefore multiple possible solutions. Then come the sheets « **Arrow** » and « **Totem** » in which, on the one hand the number of items remaining in place is more important, but on the other hand, reasoning to find the right connections is much further especially for double matches to manage and anticipate.

In the case of « **Arrow** » does not have the parts in prototypical position, that is, their side horizontal or vertical, may disturb the younger ones. A willingness assumed. Must not let to believe that the square will always be well arranged this way. An angle can be right even if its sides are horizontal and vertical. Once again an opportunity to correct the unsuitable expressions like « square on the edge » and lead to avoid confusion between diamond and square.

Finally, to reach the last degrees of progression considered, why not give only incomplete coloration of parts supplied in the beginning? A priori in the « **VI** » plug the information given is not sufficient to know the pieces already placed. One reasoning will be necessary. With the colors provided and parts available, several choices of parts are possible as contacts imposed by the shape of the puzzle will obtain limit. Tests will be required, deadlock will force them to go back and edit a made choice piece. Here we touch the beginnings of the establishment of a new type of reasoning, reasoning by contradiction. Less childish now as an activity that is relevant to detail from the college. If the area to be covered become larger and the pieces were in place not to seek among a stock of six but a twelve (sheet « **XII** »), or even more, the « puzzle » could attract even adults and require real efforts of older students' reasoning.

Finally, the creation of other figures to fill is not very complicated to achieve and it can even be interesting to ask the children to create their own puzzles. A creative time development for the puzzle appears an animal or a rocket for example, but also of reasoning. What information, what parts or colors will I give to my puzzle has a solution, but not too easy to solve. Once again, vision, anticipation and reasoning are necessary. After the design challenge and shape by coloring a number of triangles on a square grid paper to square blank parts, they will share their creations to each test. Sheet « **Creations** » is to provide support for this work by proposing two situations. Are only to be used for the first puzzle a complete set of six separate parts of the game while to the second embodiment, parts from two complete sets will be needed.

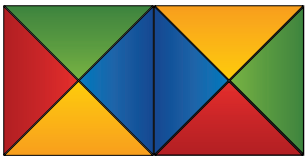
Care must be taken , however , for all this work, if it is for a very young audience , children 3 or 4 years, to provide models to achieve scale 1. The work should be limited to find the right part and to ask the right place spotted . Needless to add some potential pitfalls for reproducing model and manage different scales.
Work , one difficulty introduced at each stage.

These initial different activities have therefore, in the context of the game, aims to train and develop visual skills for anticipation hoping necessary to win. Obviously, in a mathematically and in learning situations, we can not limit ourselves to that. The ability to anticipate and envision, certainly interesting, are not enough. Work must be enriched, to make it more in line with potential areas of official programs on school and college. Puzzles are the visible part of the business but with a whole range of questions requiring schedules to leverage skills purely mathematics, particularly around the axial symmetry, and implement multiple different reasoning.

In terms of approach and the image of what was done previously during the search of pieces, each of the first work began with a question, an open question. In the first three cards, « **Rectangles** », « **Square** » or « **Rectangles & square** », it is to determine which rectangles can be obtained by using some six different parts of the game. The search can then be more or less guided and supervised age or profile of students. Each time, the ability to manipulate the colored squares be left. Indeed, at this stage, the need to physically experience is still almost absolute. We all try, often in a haphazard, asked room after room have possibly turned to place them in a position of contact that respects of color matching. With experience and training, these multiple tests, often resulting in a large number of failure, analysis, reasoning. The possible choices are then reduced by removing impossibilities. Better, after the work, must be able, mentally and without any manipulation of whether a piece can fill a space and then what position it must have.

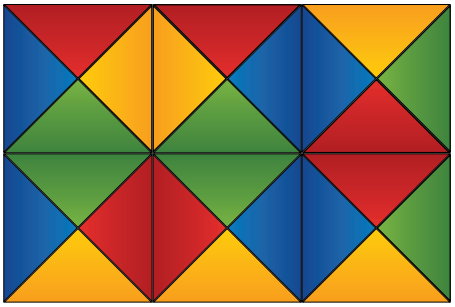
With a set of six different pieces, what are the rectangles it is possible to obtain?
A simple question that even without being formally asked, many will try to answer intuitively. We manipulate, turn, return, we try to combine triangles of the same color while seeking a regular form usually a rectangle or square, and there it is! That quickly got a first success. Would there then be other rectangles? But what is a different rectangle?
A form in which the colors are arranged differently? No, it would not necessarily much be easier to count and to enumerate. Put simply, a rectangle which will require to get it to use more or fewer parts of the game. Formulated more mathematically, we seek to create rectangles of different areas from all or some parts of a set six basic pieces. Easy to obtain a rectangle of area 1, moreover it is only just a basic piece and then just as easily we come to rectangles of area 2 and 3, 4, 5 or 6.
Interesting things and therefore to point out to our students, it is possible to form two different area 4 and area 6 rectangles because 4 can be divided into 1x4 or 2×2 and 6 meanwhile in 1×6 or 2×3. The opportunity is too good to make the link between geometric and arithmetic properties, talk about divisors and multiples or recall the method of calculating the area of a rectangle.

Rectangle Area 1

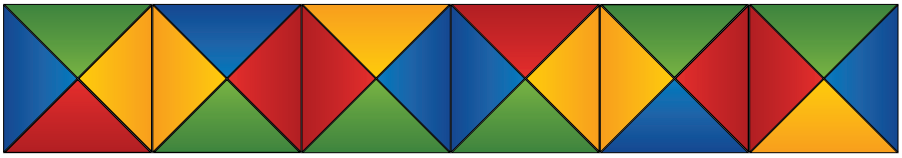


Rectangle Area 2

Rectangle Area 3



Rectangle Area 6

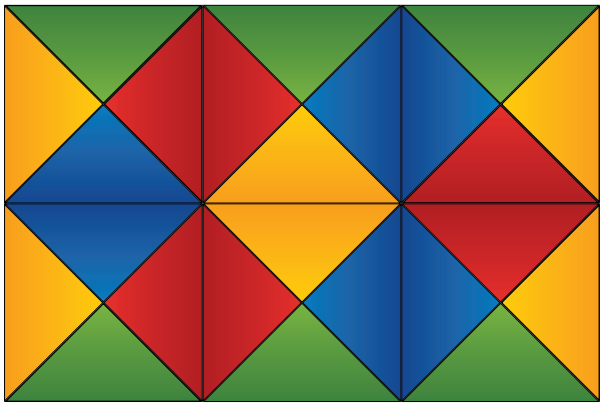


Rectangle Area 6

History to ensure that these concepts and this type of reasoning, leaving the area, being able to recover the entire dimensions of all possible rectangles with this area why not consider a new inquiry? With 24 game pieces which are all different rectangles that can be formed? After a quick numerical reasoning logically four rectangles are considered: 1×24 , 2×12, 3×8 and 4x6. The first is trivial to obtain. Others require more reflection on the investments of the various parts. More often than not, it will be the ultimate investment piece that will frustrate more than one and require backtracking to correct the wrong selection. If this task is considered too long, then why not offer more collectively in group work?

This same principle of reasoning is taken in the sheet « **Squares** » ; With two sets of basic pieces are 12 squares, look for all the squares, which is in fact only a particular type of rectangles is possible to form. 12 pieces are three squares to consider the obvious 1×1 but do not forget the 2×2 already found previously and 3×3. This basic work , suitable for a young audience can obviously be extended to research the richest of all the squares that can be formed using all parts of the game, marked the 25th included a white cross. Thus there are two cases to consider the square 4×4 and 5×5. This explains why this strange and unexpected 25th piece was added to the game to United Square offer the possibility of a game board to the perfect square form.

A rectangle or all other shape in a manner that the colors of the pieces that constitute them correspond is now something easy. However, adding some additional conditions research found an interesting character. Can we get rectangles with an element of symmetry axis or center? The sides of one and the same color? Two different colors? Many additions that completely change the steps of reasoning. In fact, it is easy to prove that a problem has a solution, just show it. By contrast , how to convince that a situation can not lead? Must then prove that the problem is not here and the richness and variety of arguments to use reasoning offer the opportunity to work so the argument is also both profound and simple.
If at first (sheet « **Borders** »), we wonder about the possible colors of the sides of a rectangle formed using a set of six basic pieces, the results can quickly arise. One border color is impossible. 10 triangles of the same color, in the case of 2×3 rectangle, or 14 for the 1x6 rectangle, are necessary in order to obtain an edge of a single color. Unfortunately, with six different pieces, only six identical triangles are available. While still other cases to consider two colors at the edges and three colors and then eventually four. The first situation is resolved quickly.



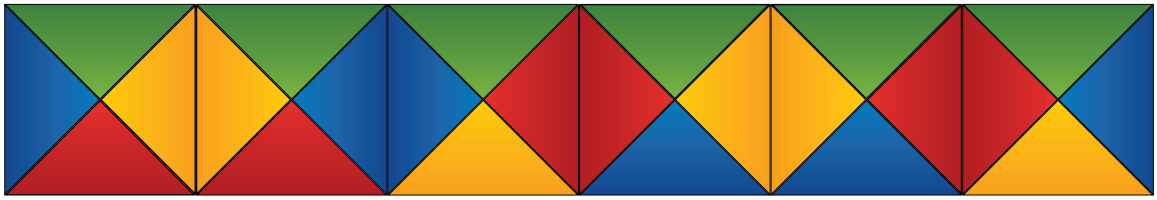
Only two colors are present on the edges

For the other two, we have to think more. Let's already rule out for possible rectangles 1×6 . To have the edges of three or four colors, the two lengths are twice six triangles of the same color are needed. It is therefore necessary to have six pieces including two colors that are the same in opposition. Impossible, only two such squares exist for each pair of colors. What rectangles now are 2x3?

It is clear that three or four colors on the edges at least one length will be affected and require three triangles of the same color. It will then remain three to use for contacts « inside ». As they are bound in pairs since only triangles of the same color can match, one of them will end up sole. Impossible! Rectangles consisting of a set of different parts, and six edges of three or four different colors therefore does not exist.

This dual reasoning, both the number of triangles of the same available color and the parity of the number of those required for domestic contacts can be generalized to all sizes of rectangles.
Why limit yourself to only using a single set of six different pieces?
Indeed, beautiful results are emerging. Unable for example to form rectangles alongside four different colors if the length and width are not the same parity...
Sheet « **Well colored** » also has aim to allow it to feel the good result.

If it slightly weakens the condition imposed (sheet « **Borders** ») certainly have three sides of same and different colors but nothing requiring the last side, the problem has a solution.



A rectangle whose three sides are of uniform color

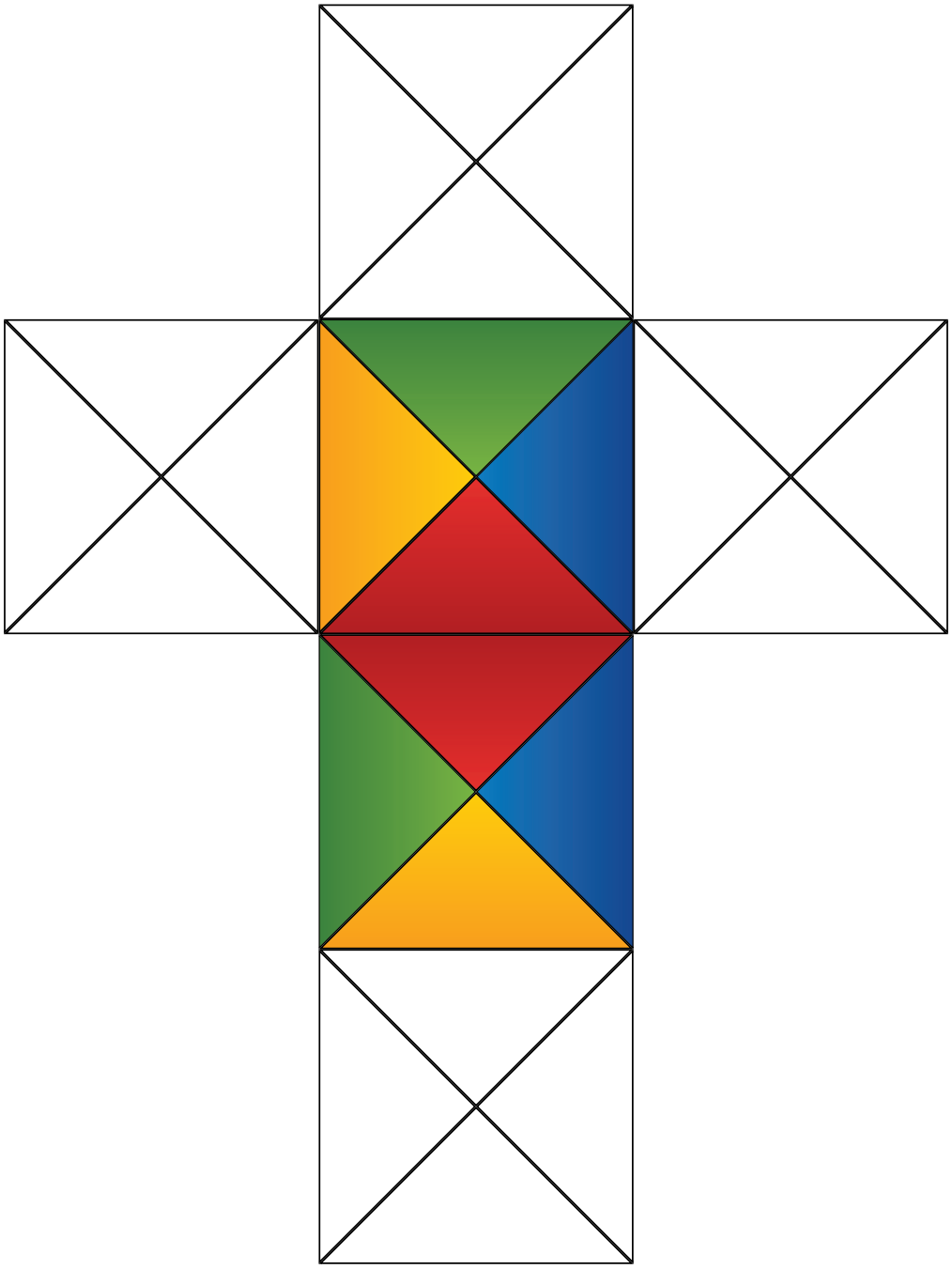
And now, if we wanted to produce symmetrical assemblies? It is quickly acquired as rectangles 1×6 or 2×3 may not have two axes of symmetry. It obviously would require that, on the same piece, a color is present twice in the opposite position. Said point being excluded, no rectangle formed with a set of six different pieces having two axes of symmetry.

Can be a center of symmetry then? Not any more! To do this would require two identical pieces for the corners of the rectangle. Impossible if we limit ourselves to a set of six different pieces. Such arguments are at the base of the sheet « **Symmetry** ».

We still have the possibility of increasing the number of items in stock with eventual duplicates or change the present area, that is to say the number of pieces to assemble or form to create. A whole range of activities therefore to jointly manage these notions sometimes visually delicate symmetry and develop various arguments. Perfect for pivotal years in school and college, right?

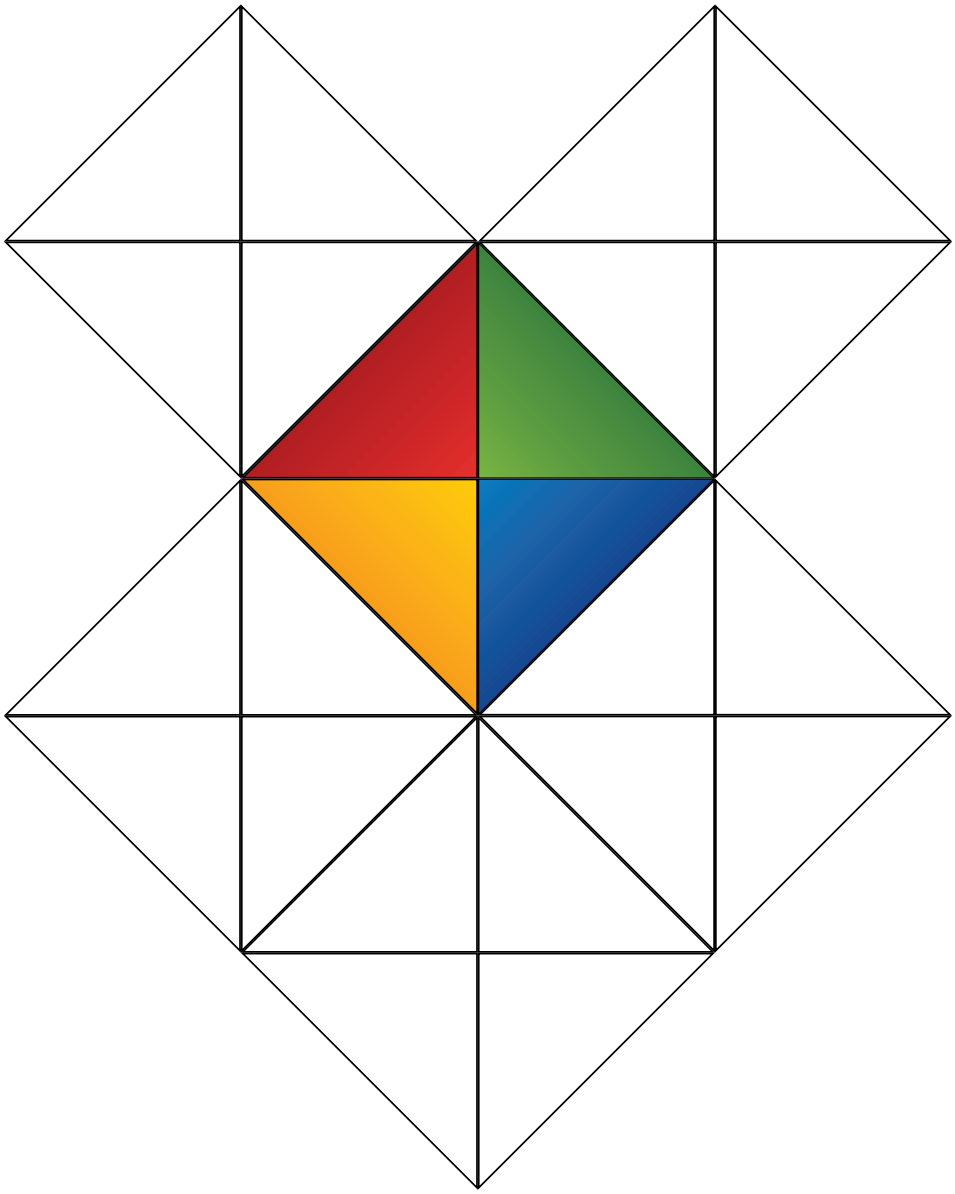
COSTUME

Complete this figure using some squares of a batch of six basic parts and in a manner that the colors of the pieces in contact match.



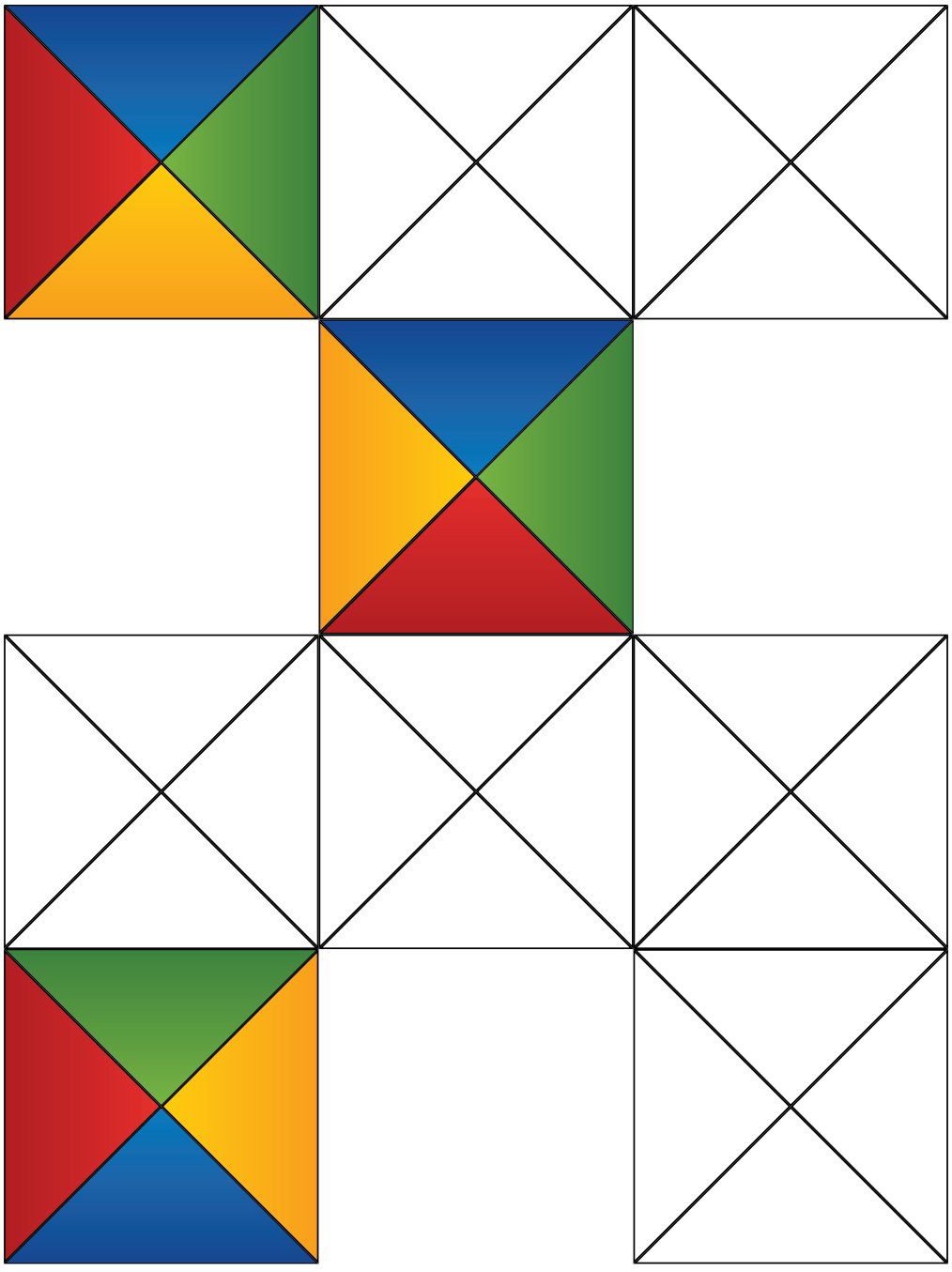
ARROW

Complete this figure using a batch of six basic pieces and in a manner so the colors of the pieces in contact match.

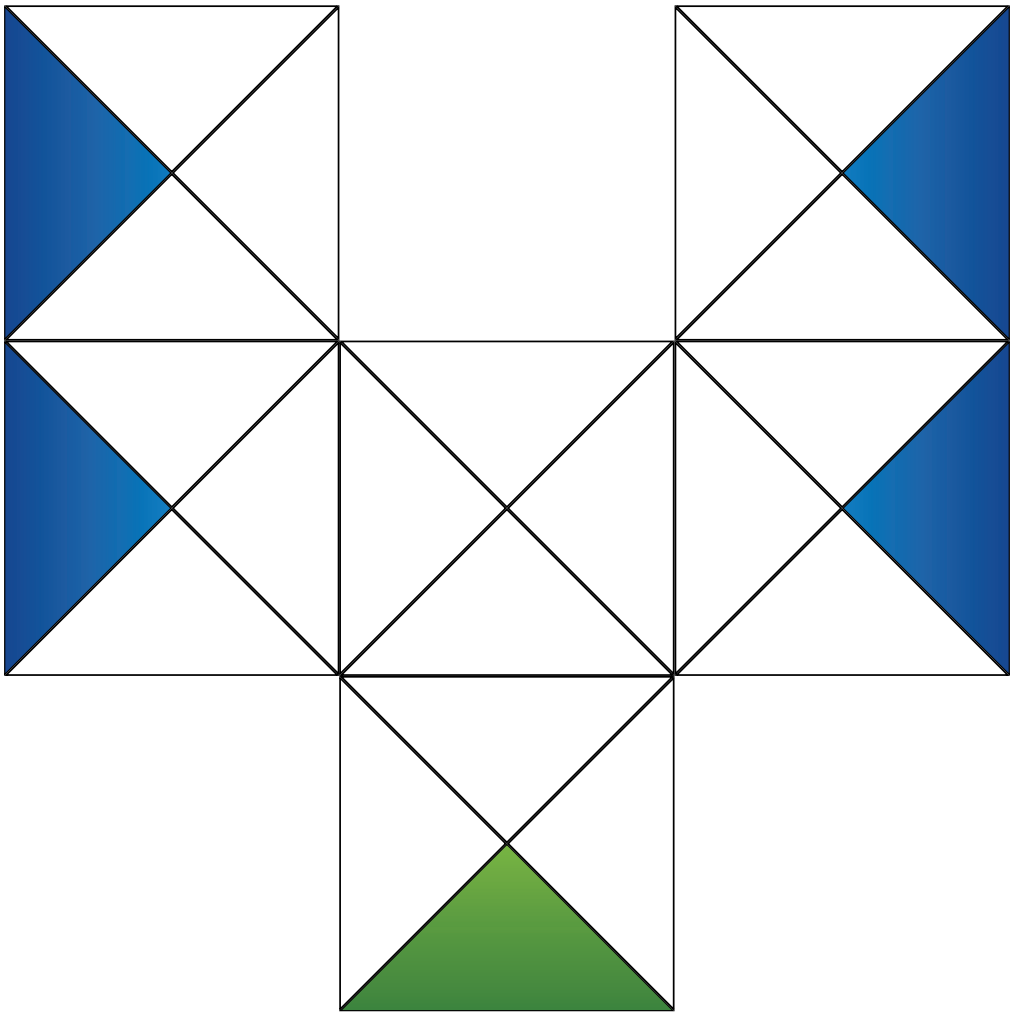


TOTEM

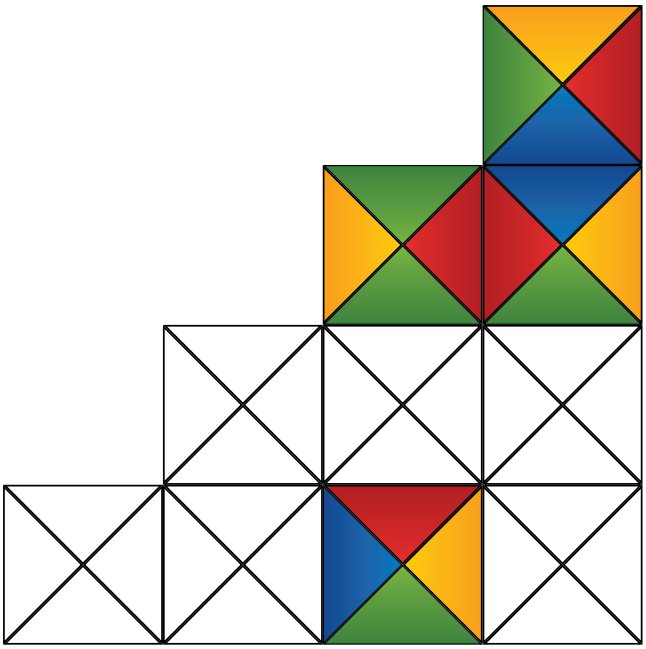
Complete this figure using a batch of six basic pieces and in a manner so that the colors of the pieces in contact match.

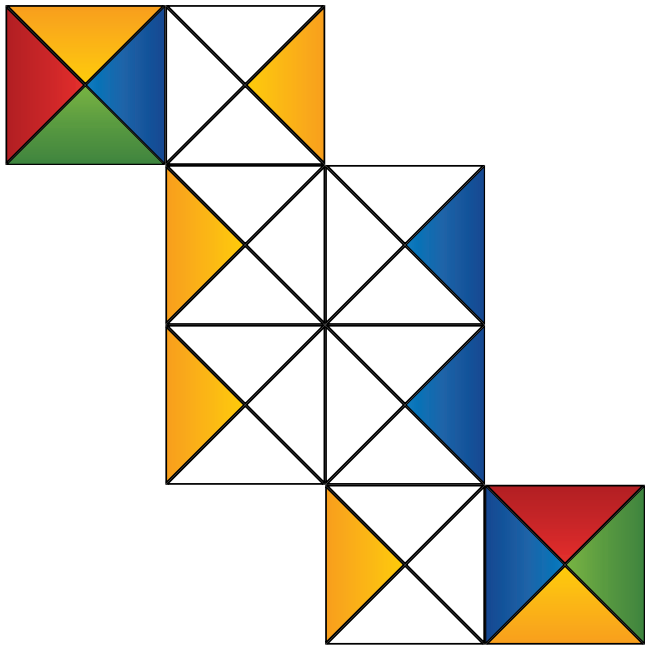


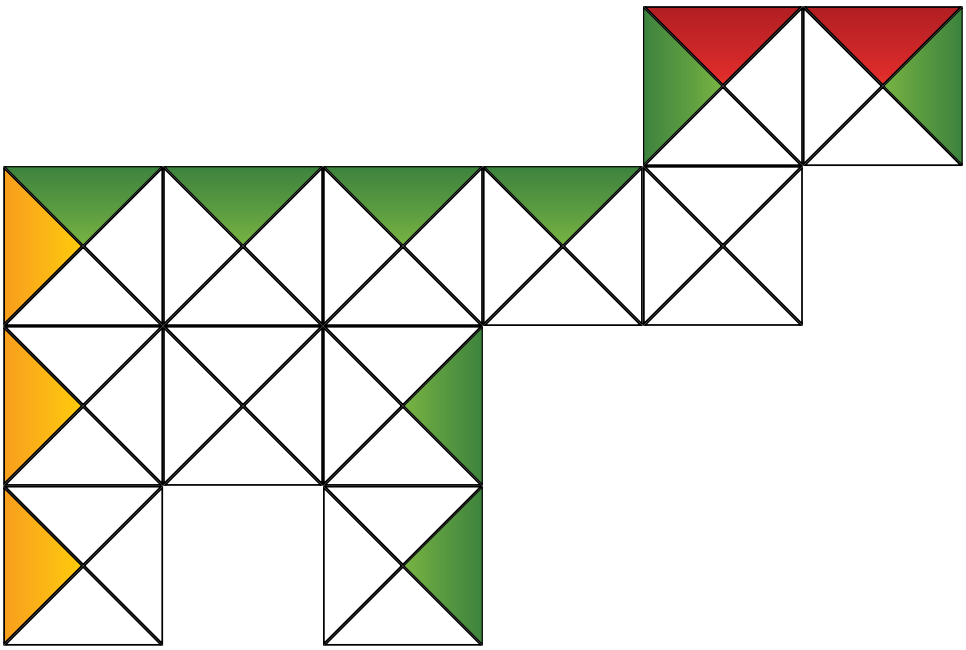
Complete this figure using a batch of six basic pieces and in a manner so that the colors of the pieces in contact match.
The colors already indicated are also to be respected.



Complete these figures using some of the pieces of two basic sets and in a manner so that the colors of the contacting parts match.
The colors already indicated are also to be respected.

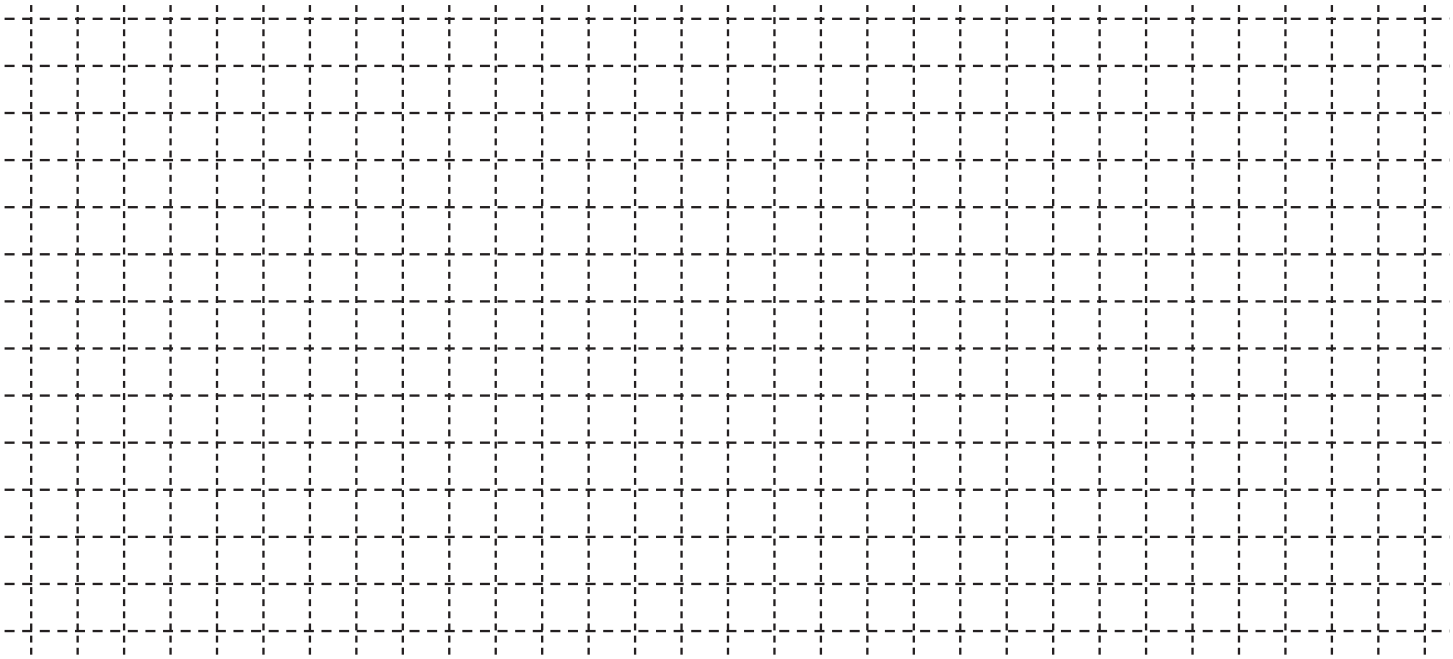




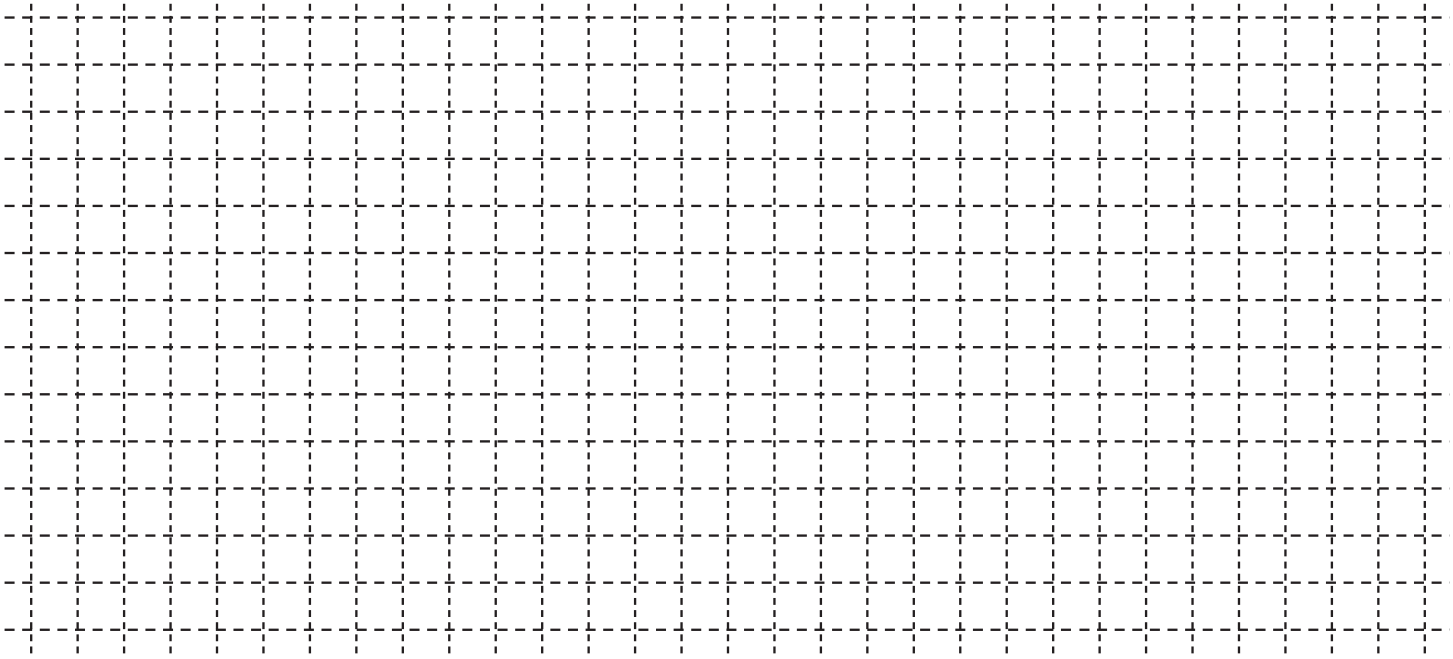


CREATIONS

Create a puzzle, vto succeed, will require the use of certain pieces of a set of six basic pieces and require that the colors of pieces in contact correspond.
All liberty is left to choose the « clues » given in the initial situation.



Create a puzzle, to succeed, will require the use of certain parts of two set of six basic pieces and require that the colors of the parts in contact match.
All liberty is left to choose the « clues » given in the initial situation.

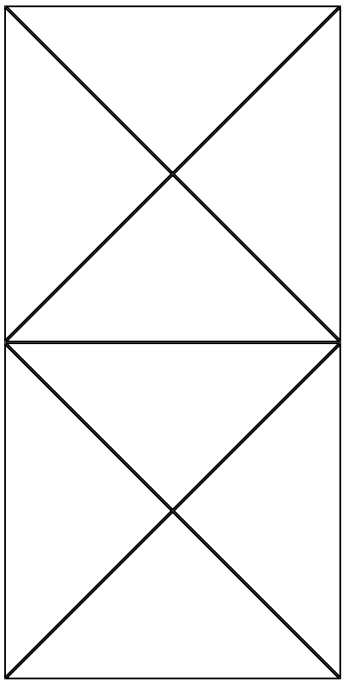


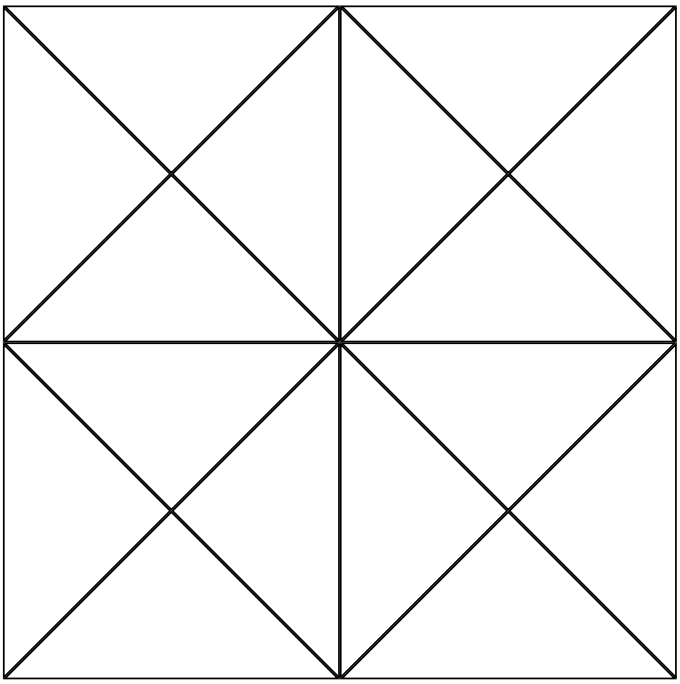
RECTANGLES

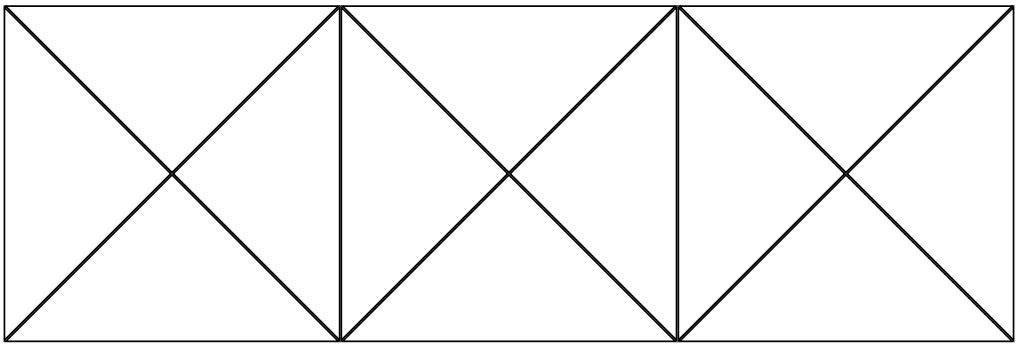
By using some of the six different pieces at your disposal, you will try to identify all possible rectangles, squares eventually, of different sizes that are possible to form.
One sole condition to be respected:

Only triangles of the same color can be in contact.

To help you begin your search, here are some rectangles to cover by certain six pieces available to you and respecting the contact condition imposed.







SQUARES

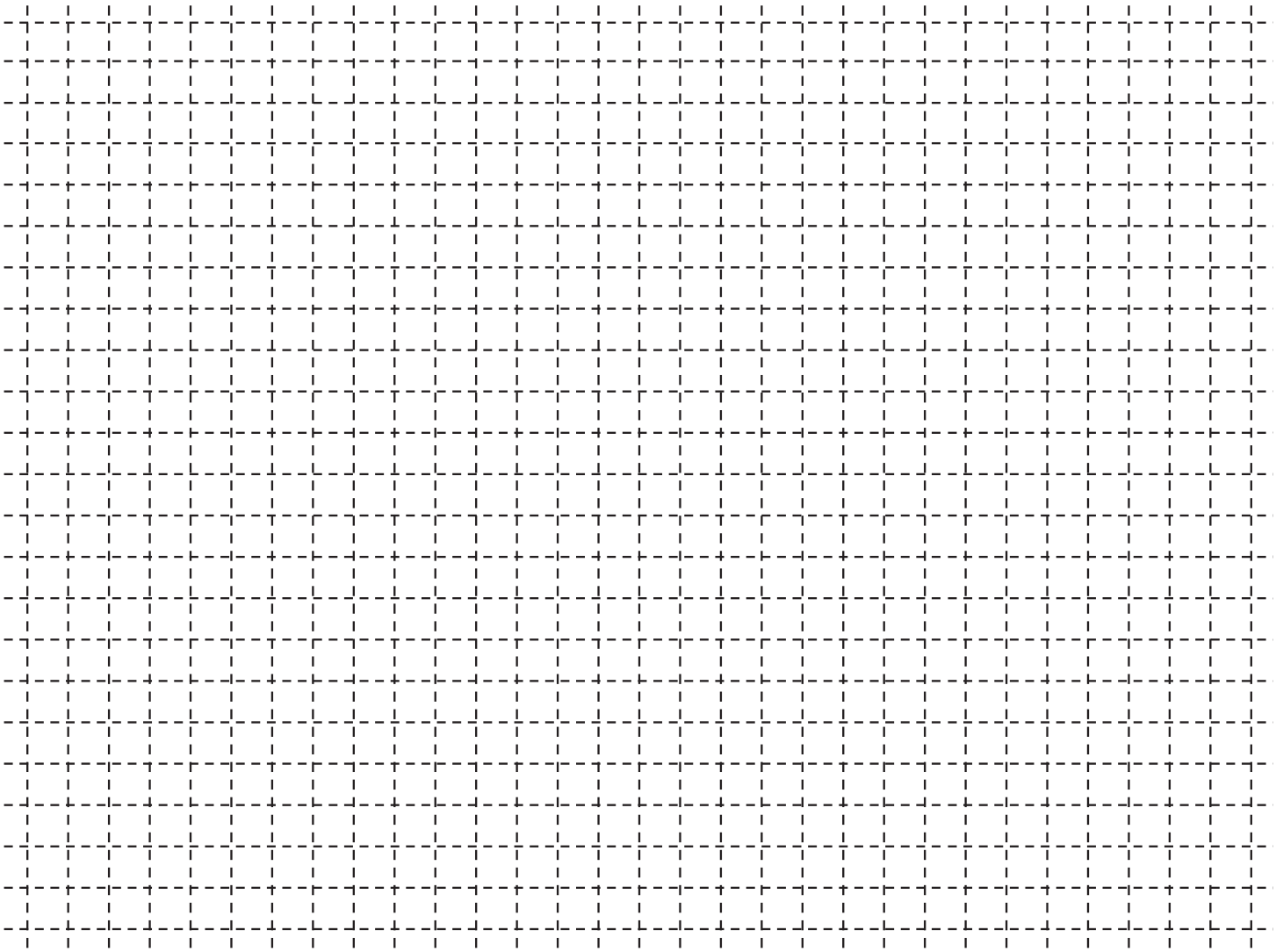
By using some of the twelve pieces, from two identical sets of six basic parts at your disposal, you will try to identify all the squares of different sizes that can be formed .
Just one condition is to meet:

Only triangles of the same color can be in contact.

At first you can specify dimensions of the square that is, in principle, possible to form.

- .....
- .....
- .....
- .....

In a second step , for each of these dimensions , try to recreate a square respecting the condition imposed.
You will introduce all your solutions in the following grid.

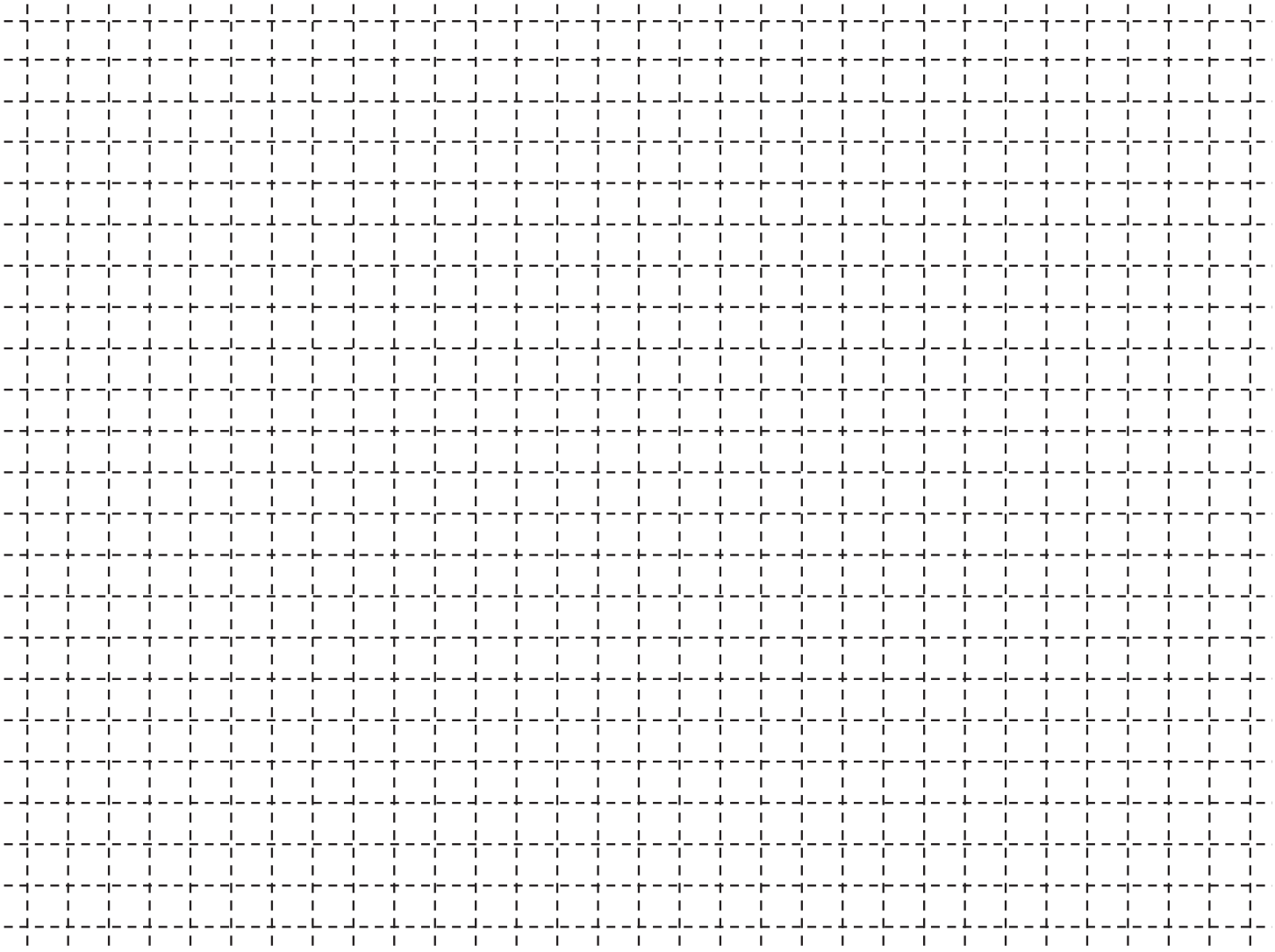


RECTANGLES & SQUARES

By using some of the six different pieces at your disposal, you'll be looking at identifying all rectangles, squares of different dimensions possibly that is possible to form. One condition is to be respected:

Only triangles of the same color can be in contact.

You will introduce all your solutions in the following grid and complete the table summary showing the dimensions of the rectangles obtained.



Number of pieces used	*Length of Rectangle*	*Width of Rectangle*

Number of pieces used	*Length of Rectangle*	*Width of Rectangle*

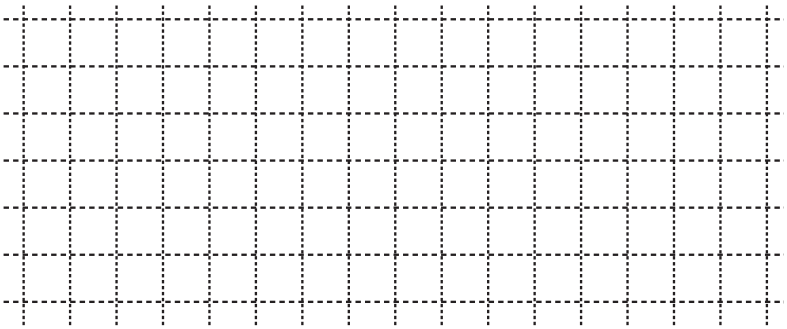
BORDERS

Using all six different pieces at your disposal, you will try to obtain rectangles whose sides respect the set color conditions. Furthermore , another condition is to be respected:

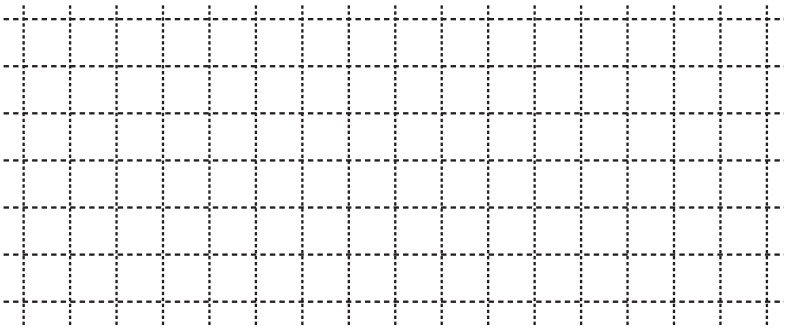
Only triangles of the same color can be in contact.

You will introduce all your solutions in the following grid.

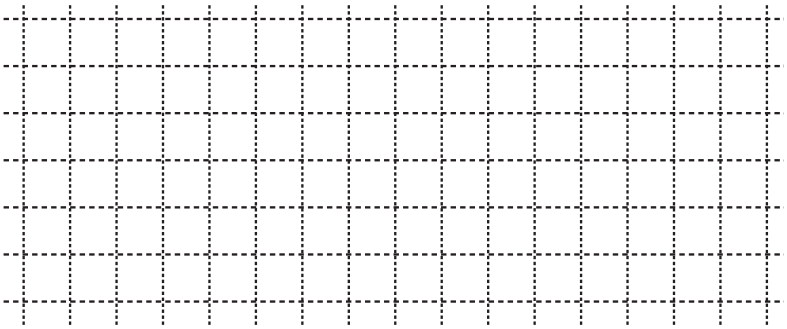
A rectangle on whose sides, the four colors are present, and none is present is a single color.



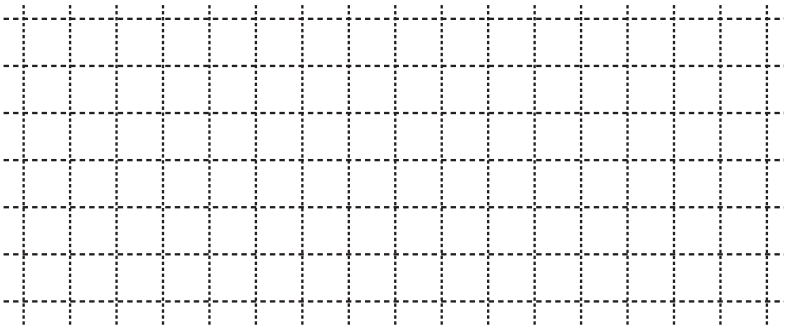
A rectangle on whose sides, the four colors are present and three is a single color.



A rectangle whose sides, exactly three colors are present.



A rectangle on whose sides, exactly two colors are present.

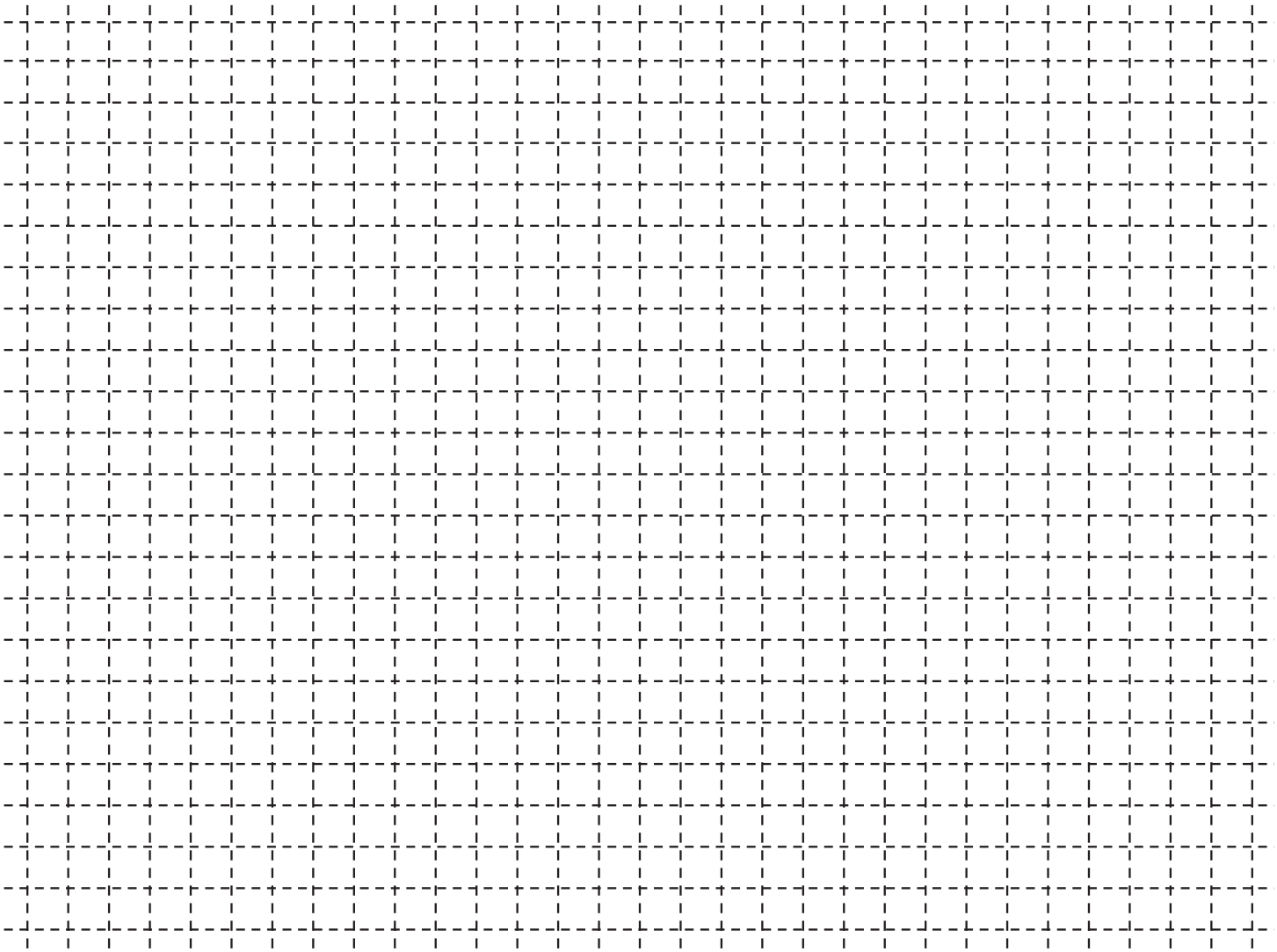


WELL COLORED

Using some parts of two sets of six different pieces at your disposal, you will try to obtain rectangles.
Three conditions are to be met:

- ***Only triangles of the same color can be in contact.***
- ***Each side should be a single color***
- ***The four sides should be different colors***

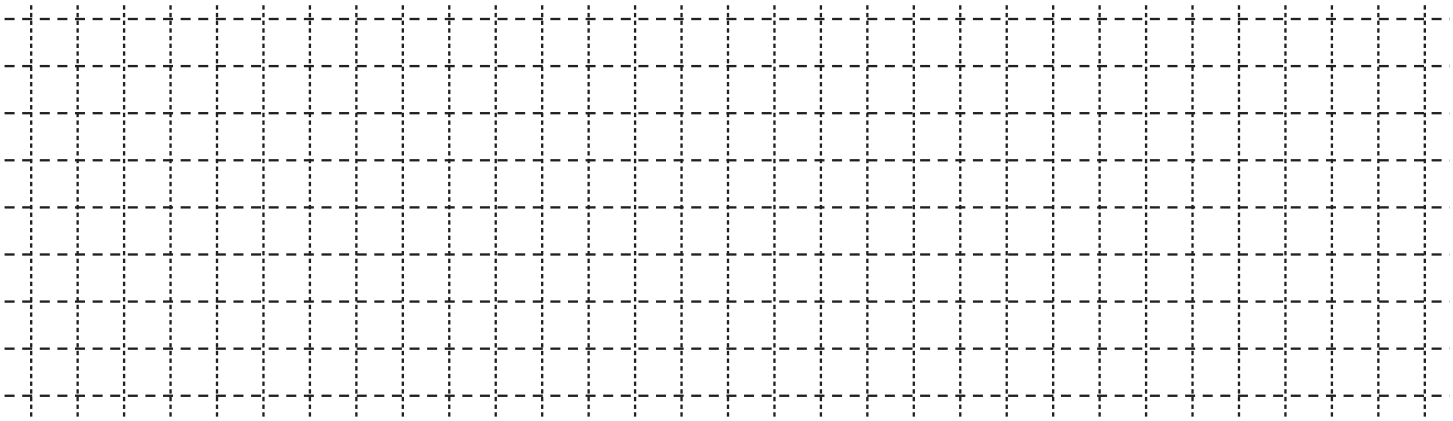
You introduce all your solutions in the grid provided or you try to explain why the solution can not exist.



SYMMETRIC

You will totally use a set of six basic pieces and juxtaposed in a manner that colors in contact are identical.

Respecting these conditions , form an assembly having an axis of symmetry.



Can we get an assembly with two axes of symmetry? Why?

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Having a center of symmetry? Why?

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Now you have two sets of basic pieces and you can limit yourself to use it only partially.

Respecting these conditions , try to form an assembly having an axis of symmetry and a center of symmetry.