UNITED SQUARE®

Educational Book 1

PIECES

PREFACE

This Educational Manual will consist of 5 chapters filled with formative activity applications of the United Square game. Each of the chapter has its own theme aimed on skills development, individual and group works on research, abstraction, reasoning and explanation involving geometry, symmetry, creativity and design.

With the succeeding chapters of this manual being still a work in progress only proves the countless other applications and challenges offered by the United Square game concept.

Educational Book have been developed in collaboration
with Stephane Robert, french Mathematic teacher.

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Before you even see the rules, the way you hold the game and even the opening of the box, it's already easy to understand that the activity is the logical construction of pieces.

We observed and noticed, that the marked logo for example: a divided square, through diagonal traces, in four triangular areas, each with different colors blue, yellow, red and green.

A question arises and a search will begin, how many such configurations are there?

Situtation open for research on multiple questions can therefore be engaged. Certainly we must determine all non-overlapping possibilities to color, four different colors, the four areas obtained by diagonal traces of a square but during this work other questions will arise. How many possibilities exist? Do i have them all? Two of my proposals are not in fact identical?

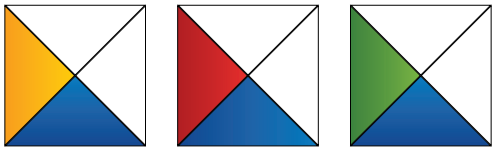
Skills employed is therefore developed or encouragements are a lot: from arguments needed to convince that two colorings are in fact identical to the discussion as to the completeness or not the solutions found. In specific terms of teaching: search, abstract, reason, explain. A very rich program!

Also have in mind that it can also add many other skills depending on protocol of the selected work, group activity, oral pooling of results or explore the use of a dynamic geometry software to present pieces found. Also have in mind, it is desirable to provide manageable supports, sets of triangles cut out of colored paper. And we allow more a wide audience, especially young children including nursery classes or students in difficulty, to start in such research without being confronted with a very high degree of abstraction. For all, a blank square sheet that can be colored will be used to list the configurations found (sheet «*Quadrichromie*»).

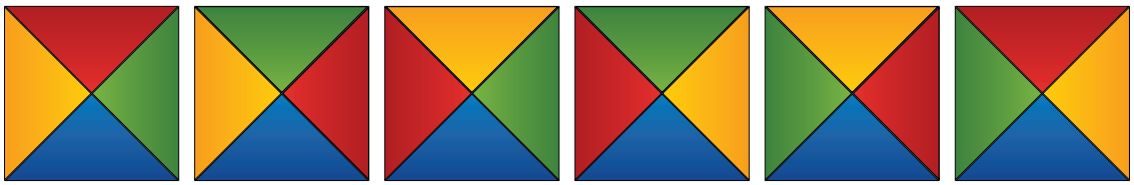
Of course, for this work, the implementation of a methodology will be necessary. A random and disorderly research might lead to obtaining similar but not identical coloring and as a consequence, difficult to identify and eliminate. To this will be added to the risk of forgetting some possibilities. Therefore, encouraged to exercise the method of organization and list each solutions obtained if they are the result of assembly manipulation of cardboard triangles. Several approaches supposed to emerge can be reproduced and generalized in other situations. They, on the otherhand, will confront particularly to compare their respective effectiveness.

Firstly, the most natural, the most intuitive, is to choose one of four isosceles right triangles and set its color. We begin by coloring the triangle at the bottom in blue.

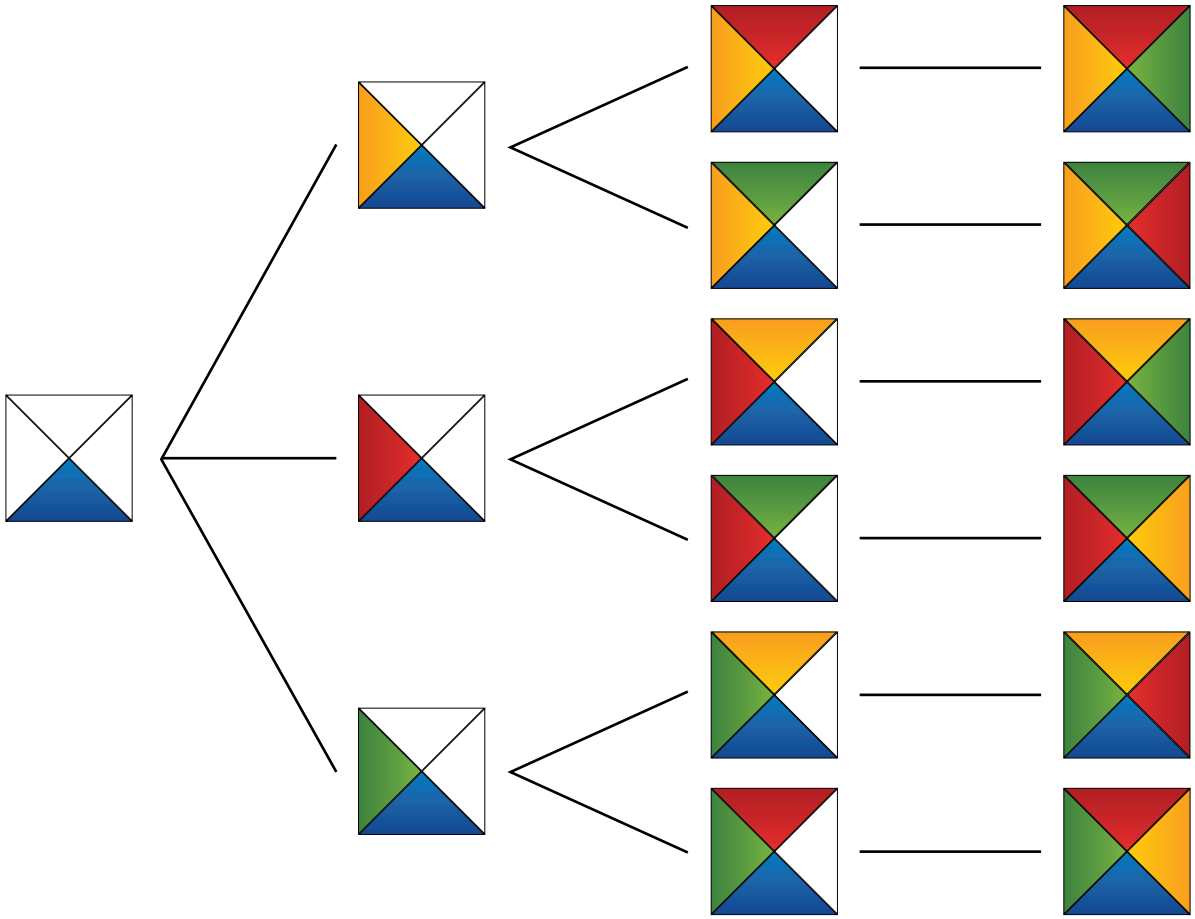
Then, by choosing a course of direction, the hands of a watch for example, we have to choose the color of the adjacent triangle. Three options are available to us: yellow, red or green.



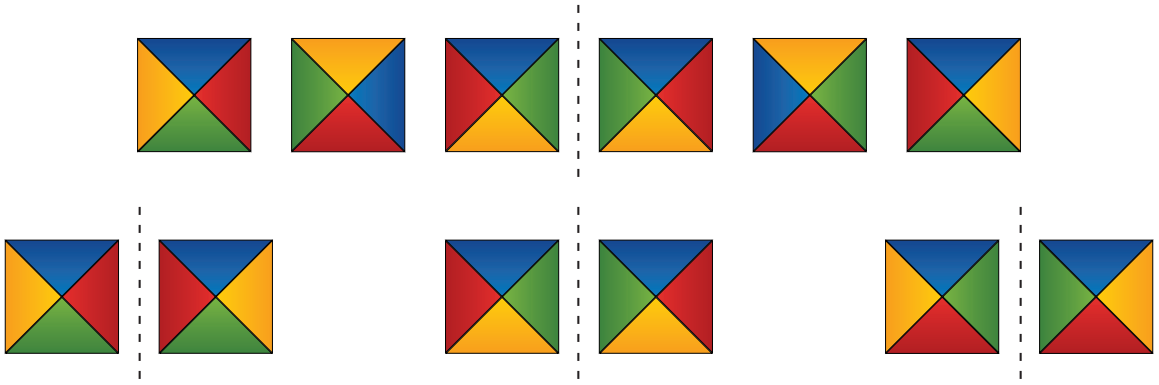
For the third area, we only have two choices to consider since only two colors are still unused. Therefore, each of the provisions, «blue-yellow», «blue-red» and «blue-green» will generate two new situations. In total, respecting the conditions imposed, six separate pieces exist.
All pieces of the United Square game will then be composed of four sets of six colored squares.



Repeating the different steps of the previous process, that is to say, by choosing to set the color of the lower triangle, here it's blue, it is possible to represent this reasoning in the form of a tree.
Thus, we find a classic form of representating situations of numeration or probabilities (sheet «*Trees*»)



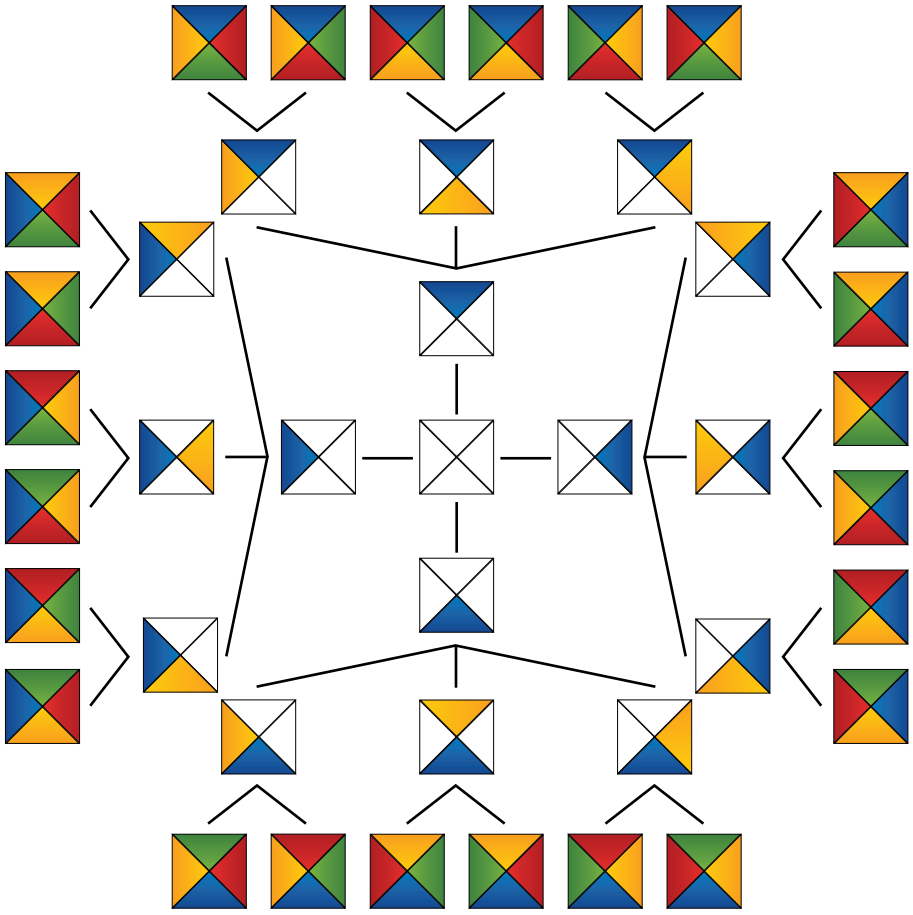
To gain even more speed, once a configuration is found, we can associate it to one which is symmetrical with respect to a vertical axis. A second solution is then immediately obtained, reducing the search time in half. The six pieces then form pairs of colored squares symmetrical with respect to a straight line.



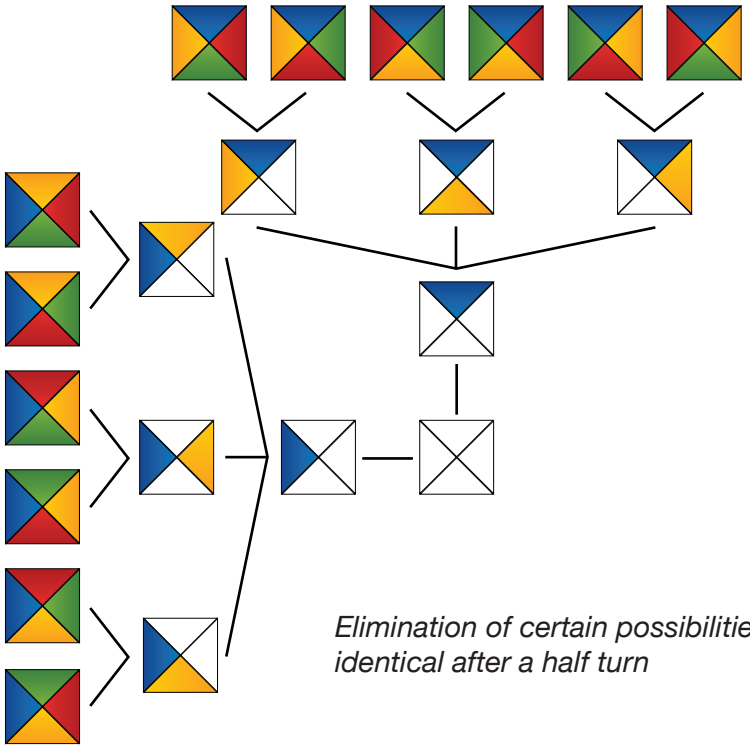
The six coloration possibilities arranged symmetrically

Another option could be considered: choose as a criterion, not to fix the position of a color, blue at the bottom, but rather a type of color and then to focus on all possible positions. The research then focuses on the enumeration of all possible color distributions. If we represent this work, once again, in the form of a tree, we have four possible positions for blue triangle and, each time, three for the yellow and finally, two for red. The last area can obviously only be green. The complete tree where 24 branches of which we have to eliminate the identical pieces but arranged differently. So what strategy to be adopted in this work of identification and elimination of duplicates?

If instead of presenting this research in the form of a tree, we instead opted for a kind of «corolla», four directions for each of the four triangular areas can be colored in blue, symmetry elements appear.
Two opposite sides, for example, are symmetrical with respect to the center piece. So we arrive at identical solutions that can be layered after half a turn. Only two consecutive sides of the square are, therefore, to be maintained, say twelve provisions.

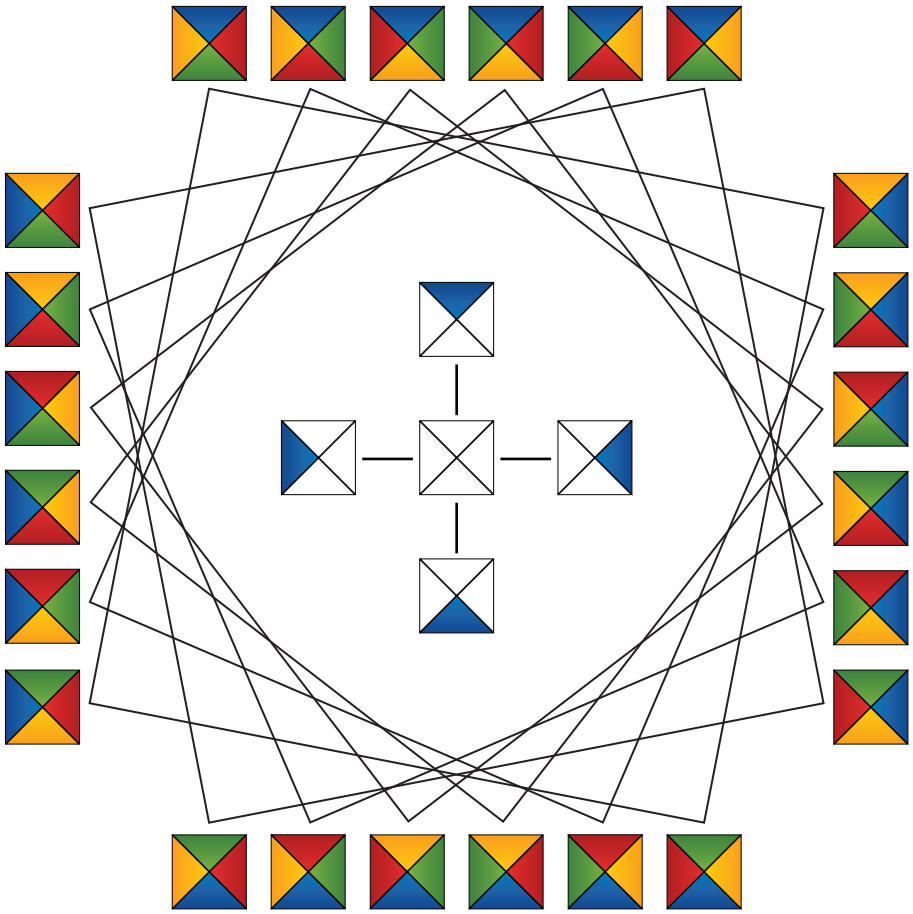


Color the first triangle of a color, four starting points, four directions



Elimination of certain possibilities that are identical after a half turn

The complete tree with 24 branches that needs to eliminate supernumerary elements (sheet «Flowerings») may allow students eventually blocked after a while of work to obtain some success. Therefore, a form of assistance to overcome the biggest possiblities of research or an eventual lack of organization on this one.
Vision skills in space, being able to mentally rotate the pieces, replaces the original purpose of research. However, positive, the success of all, lead to six different pieces that make up United Square, is the objective. Finally, without forcibly noticing the existing symmetries, it is possible to proceed simply by recognizing the identical pieces.



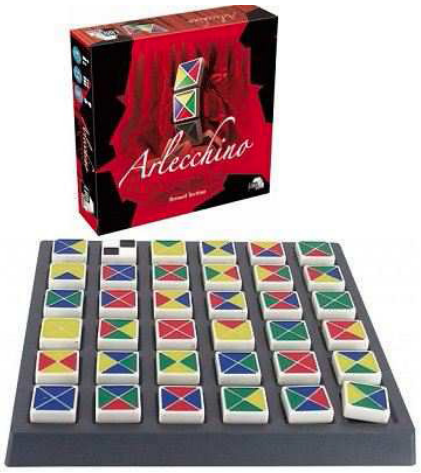
Identical pieces are connected by a segment

An original way to ensure that the logic of construction of pieces just like all the previous algorithms are well acquired could be made into a small game. All the 24 pieces in the box of United Square are placed freely in an opaque bag. One of them, selected at random, is pulled out without being revealed. By observation and possible handling of 23 remaining pieces, must redraw the piece put aside. An opportunity to let the children practice working as a group and let them discuss and contend about their strategies, their choices.
If the group activity repels, the sheet «*Eliminations*?» incorporates the principle suggested for two successive draws of a piece. The dynamic aspect and the richness of exchanges are lost in favor of a more traditional individual activity.

To get faster the number of existing pieces, a simple numerical reasoning would be enough. Anyway, it is also very close to the «Constructivist» approach we have just seen. Once a first triangle's color is set, we no longer have for the other player, the one at his left for example, just three choices of colorings possible. Then two others to the adjacent area and a single and unique for the last triangle. In the end, therefore can only exist as 3 x 2 x 1, or also «3!»or «3 factorial» different pieces. A quick and interesting approach of enumerating situations which unfortunately does not actually permit us to build them effectively. However, this type of reasoning is much fully interesting in research situations certainly close but far more complex.
Modify then a few initial terms of the research and then ask ourselves the consequences. We also have a simple way to provide a new task for quick students, also now in activity while in pursuit of slower work or ensuring that the previously proposed strategies can be reinvested. When the number of considered cases strongly increases, the effective construction of the pieces becomes long or tedious. The enumeration of the possibilities can result in a first phase of research can be more than enough.

What will happen, then, if the initial conditions are slightly modified? If the initial parameters are extended?
For example, why not allow the colors to repeat on the same square? We take note that the pieces used in many games are obtained from these small variations.
Citing as examples: «*Quads, La guerre des moutons*» or Arlecchino. Sometimes the choice made is to increase the number of colors used - «La guerre des moutons» - or opt for forms of varying symbols, horizontal or vertical lines - Quads - at other times are the repetitions of the same color which are allowed just like in Arlecchino. It is interesting to note that all existing pieces and verifying the initial conditions are not necessarily preserved.
The reasons are diverse. Sometimes, they are economical and are used to to limit very large number of pieces.
For other games, they are strategic to avoid imbalances between players or prevent some deadlock situations that can be caused by some configurations.

In «La guerre de moutons, by Philippe des Pallières and edited by Asmodée and himself, one color per player is presented where we add «forêt» (forest) and «bergerie» (sheep-pen), four areas to color by choosing among six colors.

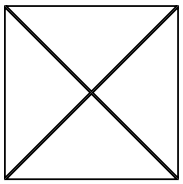
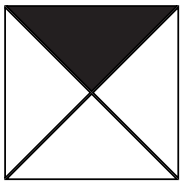
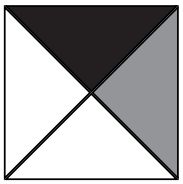
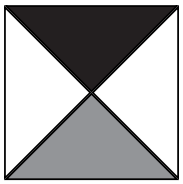


For Quads, a blocking game by Kris Burm of Gigamic, pieces have two types of hatchings and two colors but not present simultaneously on the same piece.

In Arlecchino, majority game by Bernard Taviatan and edited by Lud Editions, four areas to color with four shades with possible recurrence.

Treat for example a relatively simple situation though very rich mathematically and not without historical interest. We limit ourselves in this new research coloring the four triangular areas with colors chosen among only three possibilities, blue, yellow and red. Less choice of colors than in the case of United Square but the possibility of recurrence of colors given. One finds a very famous recreational mathematical puzzle from the late nineteenth century, the square MacMahon, named after its «inventor», the British mathematician Percy Alexander MacMahon.

The reasoning will remain close to that used previously but with minor changes in the choice of presentation. To begin, we identify different types of pieces to be considered depending on the number and distribution of the possible colors herein.

1 color	***2 colors***			***3 colors***	
					

In a second step, we need to determine the number of possible color choices:

- If only one color is present on a piece, the number of possibilities are obvious, three like the number of colors available.
- If two colors are present, three distinct cases arise.

  Consider the first. We must first of all choose a color among the three to color the isosceles triangle.

  Then, determine a second remaining three but this time, more among two or else we fall in the case of a single color as previously discussed.

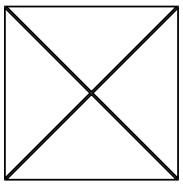
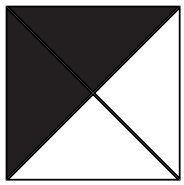
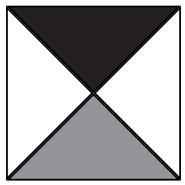
  To summarize, two colors to choose from a selection of three and whose order of choice is important in enumeration of its own mathematical vocabulary,

  an arrangement, note here A2/3 and equal to 3 x (3-2 + 1) or 3! / (3-2)!

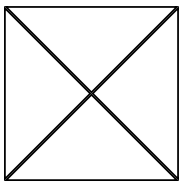
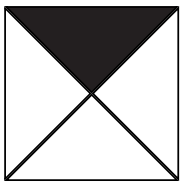
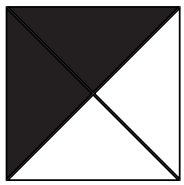
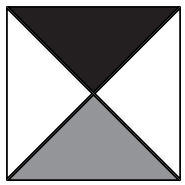
  One occasion, when such work, introduces new concepts, arrangements and combinations, appropriate to a high school program and allow them to discover in a concrete, rich and playful situation

  Of course, this work of enumeration is exclusively reserved to secondary students but can be proposed earlier.

If three colors are present, a similar reasoning is to be used.
In the end, treating all the possibilities, we can add a line to our table.

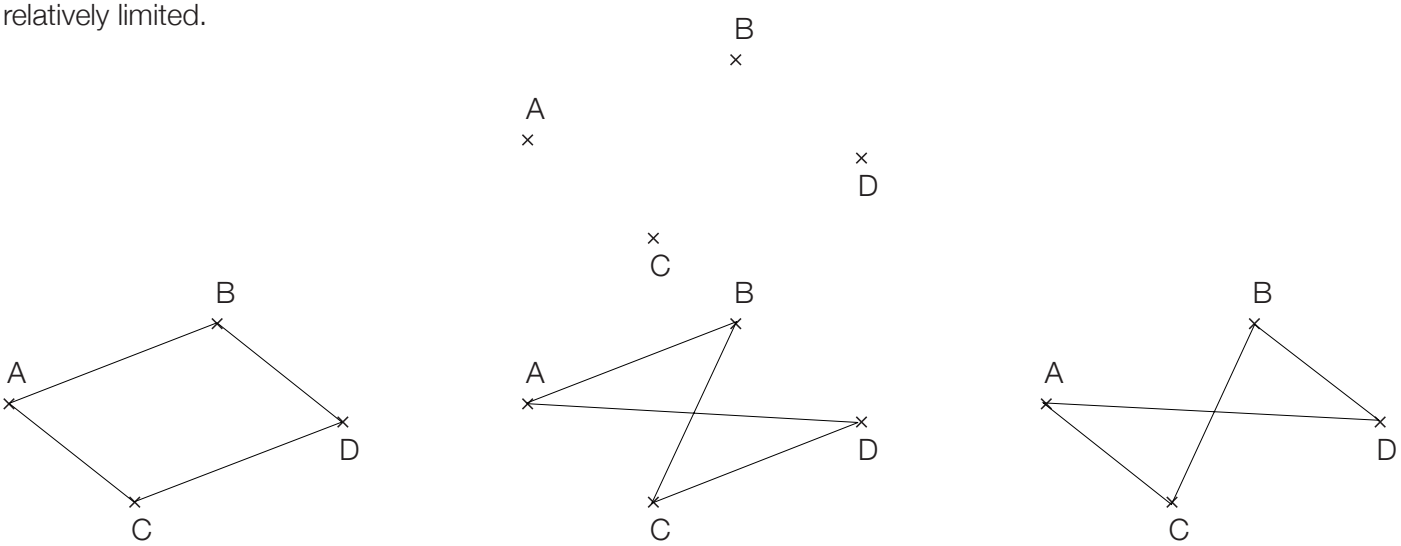
1 color	***2 colors***			***3 colors***	
					
3	***6***	***3***	***3***	***6***	***3***

Finally, what remains is to eliminate redundancies, that is, the parts which appear repeatedly in different positions after eventual rotations. It is that necessary to build these solutions then by observation, eliminate some? A simple and effective strategy but for a less longer and tedious besides with the risk of leaving some duplicates.
In the case of single-colored pieces, there is no risk of recurrence. As against, for two opposed by the top colors, a quarter turn to switch from one configuration to another may already be counted. Our count has doubled the number of real solutions. It is therefore necessary to correct our table and conclude on the existence of 24 pieces.

1 color	***2 colors***			***3 colors***	
					
3	***6***	***3***	***3***	***6***	***3***

After the work of enumeration and discovery of six different pieces that make up the United Square game, it is essential, always a possible prerequisite to discover the rules of this game, manipulating the maximum colored squares. We must be able to recognize and differentiate whatever the position. On this purpose, nothing better than to manipulate, to turn in all directions. Different activities are then considered, simple reproductions of colorful motifs, through the finer observation of differences between two pieces, to begin with, the search for a possible name of each square.
It will quickly need us to communicate the choice of a piece among six at our disposal. How? Assign a name to each, for example in the form of letters from A up to E, and indicate the selected piece? A means undoubtedly effective provided that the corresponding letter-piece has been known, that is far from easy. Another possiblity, more universal, would be to choose a course of direction and then name the colors in the order in which they appear. An interesting parallel is then to do with the geometry and how to name polygons: choose a direction of rotation and name the tops encountered in the order of their appearance. Obvious? Not so in view of the difficulties often encountered even in secondary school when it comes to naming a parallelogram and by mistake we find ourselves with a crossed quadrilateral. The sheet «Namings» proposes, the practice to avoid this kind of unfortunate confusion, to work on the names that is possible to associate with pieces of the game with their color. A wide variety of tasks is actually swept to address the multiple facets and subtleties of this notion: name a piece, with the name search for the corresponding piece and draw or be able to determine all different titles different that can be attributed to the same piece. This work may be proposed, a choice, with or without the effective manipulation of colored squares. In this second situation, the fact of having almost mentally rotate the pieces makes the work more difficult, but much closer to the subsequent game situations in which it will anticipate possible positions without multiple experiments and rotations.

An extension or in final completion, it would be quite interesting to perform the same work with polygons. Quadrilaterals also lend themselves perfectly because they generate a sufficient number of different names but relatively limited.



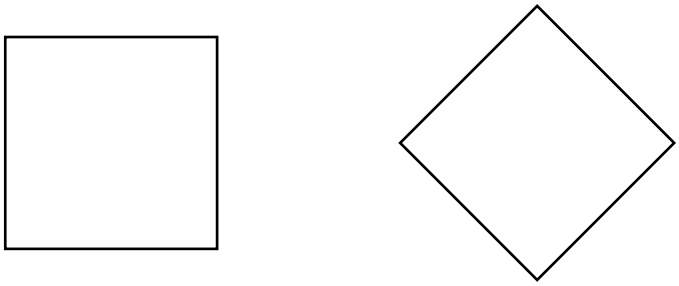
Four points A, B, C and D distributed identically but different ways to connect and logically strong to name the quadrilateral obtained.
For example ABDC has nothing to do with ABCD or even ACBD.

To continue to improve this knowledge and understanding of pieces, two types of activities are still proposed Across the sheet «***Permutations***», one may wonder that by a succession of simple inversions of two neighboring colors, it is possible to pass from any colored square to all the rest. However, with a single inversion, we can reach only four pieces among the six different existing.
We chose to represent it in the form, classical in mathematics, a graph. A graph that is not complete, since each piece is not connected to the other five, but only to four. On the other hand, we have limited the work requested, not the creation of this graph, but the identification of swapped pairs of colors. Accordingly, an important observation work and anticipation must be done.
Finally, to continue differently and start further towards the development of skills actually needed in the game, are various activities of the sheets available in the series «***Harlequin***». A common point to all these tasks: figures are reproduced with the equipment available. In the sheet «***Harlequin 1***», the patterns are obtained using as a whole a set of six different pieces. Then in «***Harlequin 2***», two sets are available and must extract the colored squares necessary.
Finally, «***Harlequin 3***», is oriented more towards work, free in principle, creating new patterns. It is to be noted and kept in mind that progressiveness is underlying this work. First of all, the patterns are basic and distributed classically, that is to say, the sides of the squares remain horizontal or vertical and in the extension of eath other. It is only when several figures have been reproduced that difficulties arise. First appears the non-continuity of the sides, then the non-prototypical position of the squares.

However, the pieces remain square and do not become miraculously diamond-shaped. The opportunity is then given to attract the attention of children, particularly the youngest, on a fundamental and oftentimes sensitive issue: it is not because a form is not typically arranged that its nature changes.

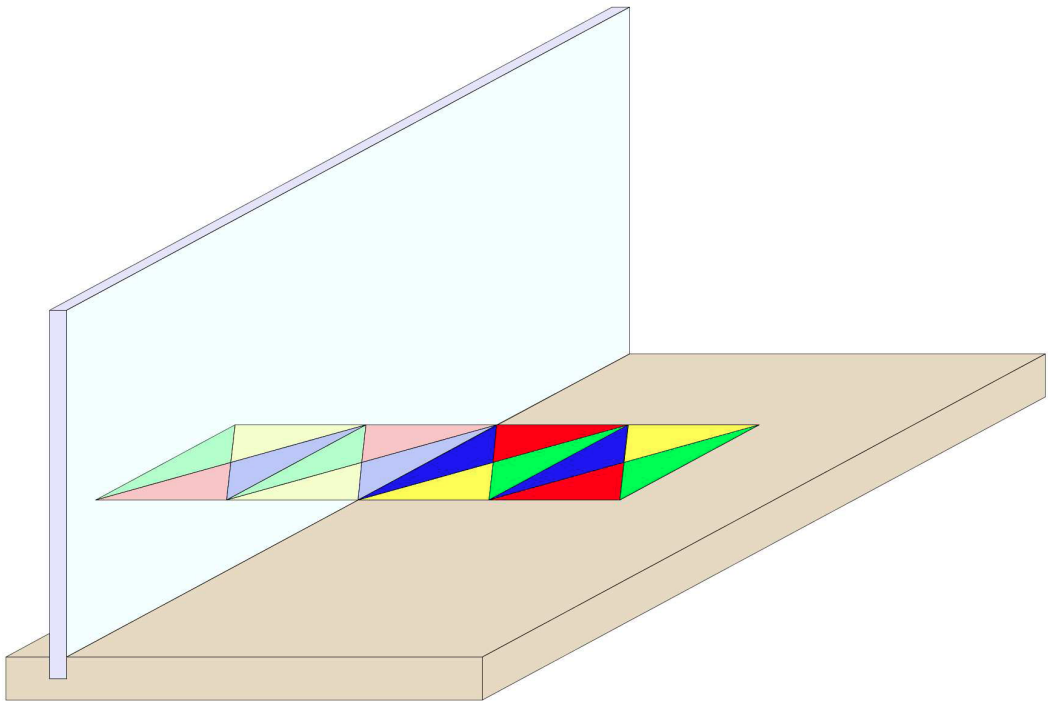
We then correct the improper expressions of the «square on the edge» style and avoid the confusions between diamond-shaped and square.

A repetition or a focus on the definition of each, square and diamond-shape, just in time.



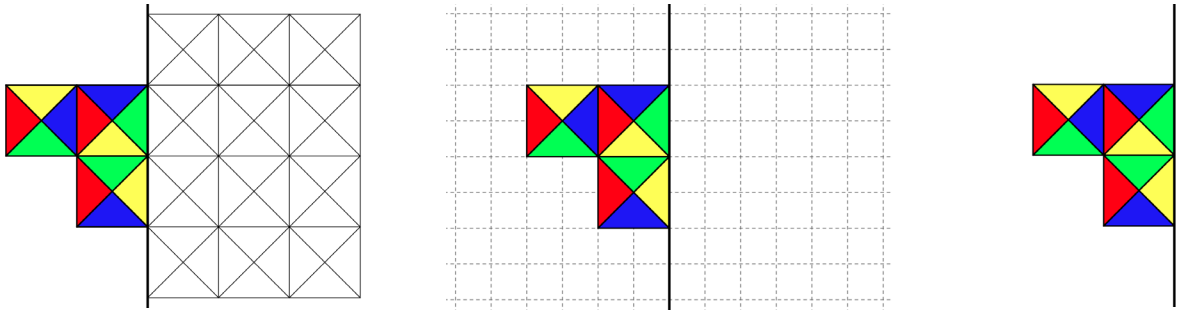
Two squares and not as often incorrectly thought of, a square and a diamond-shape

In a second step (sheet «***Harlequin 2***»), scale models reproduced varies from an animal to another.
Far from being a design flaw, it's a real desire for these differences are certainly not very difficult to handle for an adult, but in fact, much more difficult for children.
It may also be desirable for very young students (4 or 5 years), to provide the model for the sheet «***Harlequin 1***» with exact dimensions of United Square game pieces for them can instantly cover figures to be reconstructed.
A less difficulty is then managed.
As for «***Harlequin 3***» after creating a new pattern, each will design. Two levels of difficulty are also considered: coloring of the support on which blank pieces are already apparent (6 or 7) or the use of a grid from which the contour of the pieces drawn beforehand (9 or 10 years).
In conclusion of this series, it may particularly be motivating and interesting to propose the exchange of creations in a class or group with instructions to test them.
On one hand where there will be verification that the designer did not make any mistakes and also the development of the variety and the originality of the realizations.
Without a doubt, a treasure of ideas and a potential bank of new media to be reused.
Finally, an almost identical work is proposed in the sheet «***Reflections***.» The opportunity offered here in addition to work or to introduce the concept of axial symmetry.
The mirror reflects the pieces and new figures are formed.
This principle included for each pattern obtained, must first locate its axis of symmetry, then use the right pieces, reproduce one or the other half of the patterns.



Is to provide for this activity in addition to a set of six different pieces, a mirror placed vertically.
You can easily get in hardware stores reflective paper
stickers that are once attached to a vertical support will be enough.

For this task we can consider various ways: simply in solo, in team of two's to reduce the amount of materials needed and provide very enriching moments of exchange or in the form of competition, in that or of the teams that will succeed in reproducing in maximum of figures during a time period.
Finally, it is quite possible to leave some free time to create new designs in corresponding to sheet «***Harlequin 3***» or reproduction from a grid of those proposed by showing the axis of symmetry, that is, the position of the mirror.
Without the desire to walk into a hardware store or having no means to buy the mirrors necessary, the series of sheets «***Sym***» and «***Sym'***» will be able to overcome these material difficulties.
It will not be here that the mirror will complement the figure intended to reflect the construction image.
An assembly is provided and is to be completed for that or the straight plotting axes of symmetry.
Here is a work purely manipulative, very progressive and highly evolutionary, which easily allows the trials and errors quickly corrected.
To attain the maximum annexed difficulties and focus only in the understanding of the symmetry, that is, to make the manipulation as easier as possible, ideally is to provide figures to complete in the format of the pieces.
It simply then needs to directly arrange the attempts on the sheet provided: no figure to reproduce, no problem with scale managements. Slight enlargement of the supplied samples must be considered.
Finally, it is permitted to complete the task by the tracing and coloring of solutions found. A first step is thus made towards the construction of a symmetrical figure.

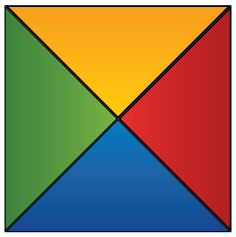


Three options to gradually move the position of pieces in the actual construction of a symmetrical figure with respect to a straight line: the simple coloring of a template, the layout of pieces using a grid, the blank sheet highlighting the possibility of simply extending certain segments.

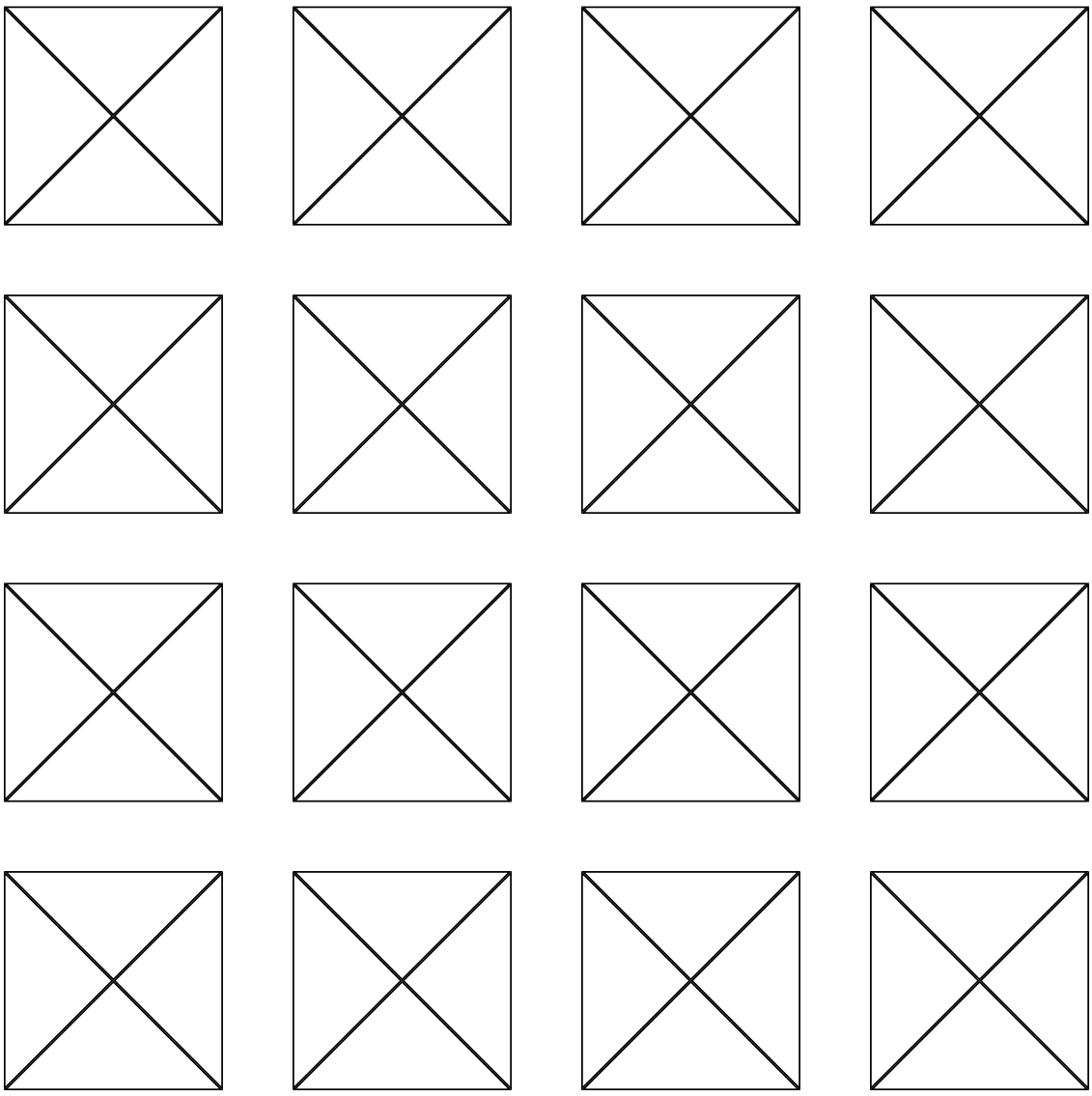
Regarding the evoked progression, the sheets «***Sym***» leave some pieces to place among a limited choice to move towards a wider selection and the axis of symmetry changes the classical positions, vertical or horizontal lines, to a more delicate oblique dispositions.
As for the sheet «***Sym'***», the aim is to focus on constructions of figures having two perpendicular symmetrical axis and thus actually implement the notion of center of symmetry.
The observation of the effect on the initial motive for the dual symmetry leads to the analogy between the composition axial symmetry of the perpendicular axis to the central symmetry of the center intersection of two lines.
These observations may then introduce this new knowledge. These are just some examples of figures. Some children may need more time, more practice to fully master the concept of symmetry.
In this case, no difficulty to create new situations.

QUADRICHROMIE

We start with a square and draw its diagonals. Four isosceles right triangles are thus created, triangles that will be colored with all four colors present and different, blue, yellow, red and green.
Example, we can get the first disposition

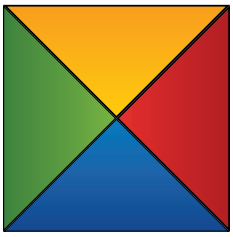


In doing so, how many different pieces can we get?
Each of your proposals will be shown below using the squares provided.

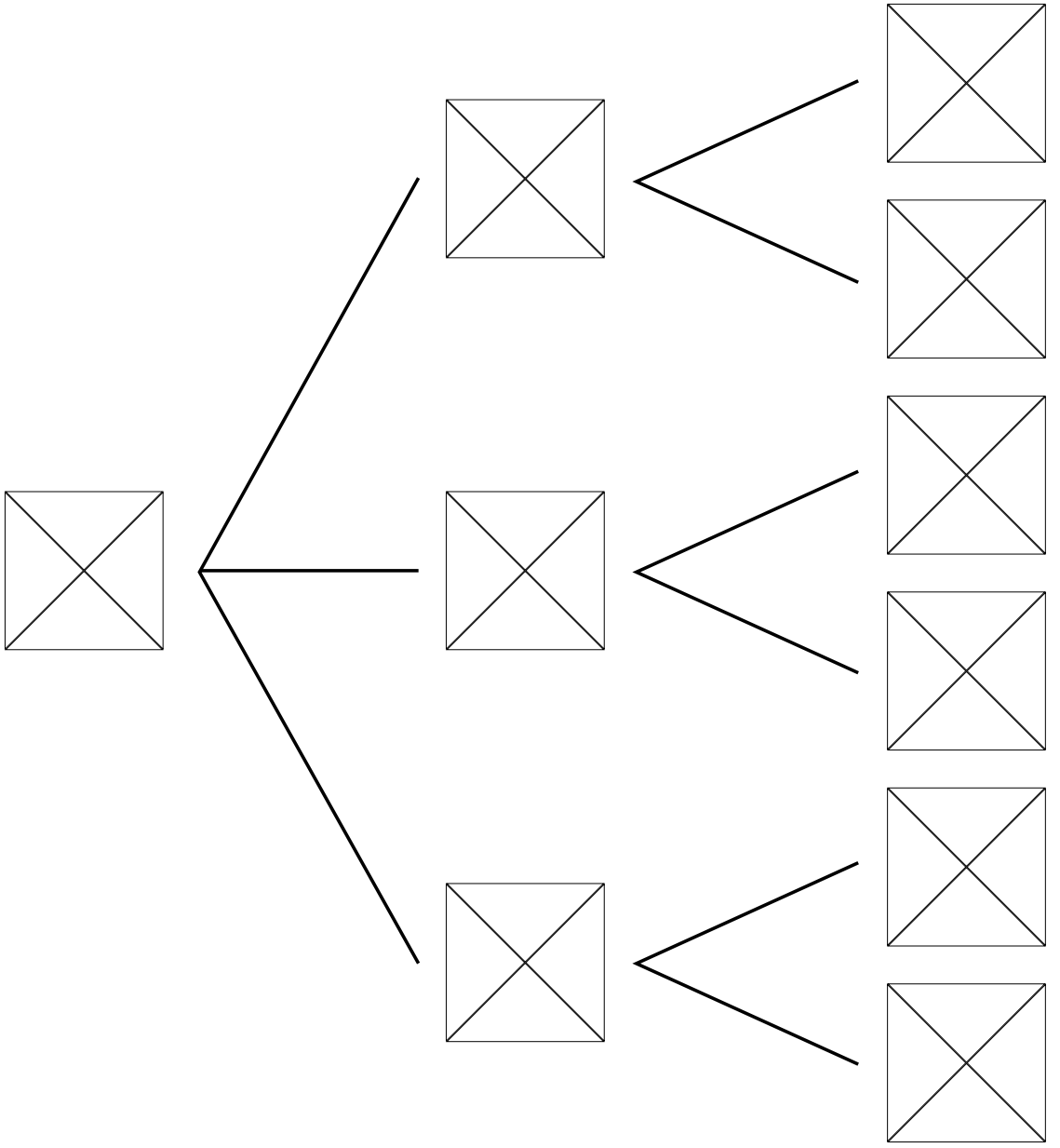


TREES

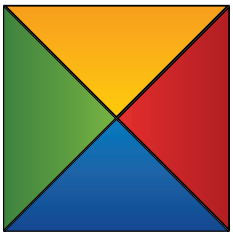
We start with a square and draw its diagonals.
Four isosceles right triangles are thus created, triangles that will be colored with all four colors present and different, blue, yellow, red and green. Example, we can get the first disposition on the opposites.
By following this rule, we want to know how many different pieces are possible to obtain and not to forget any.
For this, we will logically proceed: attaching a first triangle and continue coloring the same color then all other options to his left and so on...



By following this approach, color the possibility tree below to get all the different pieces corresponding to the initial conditions set.

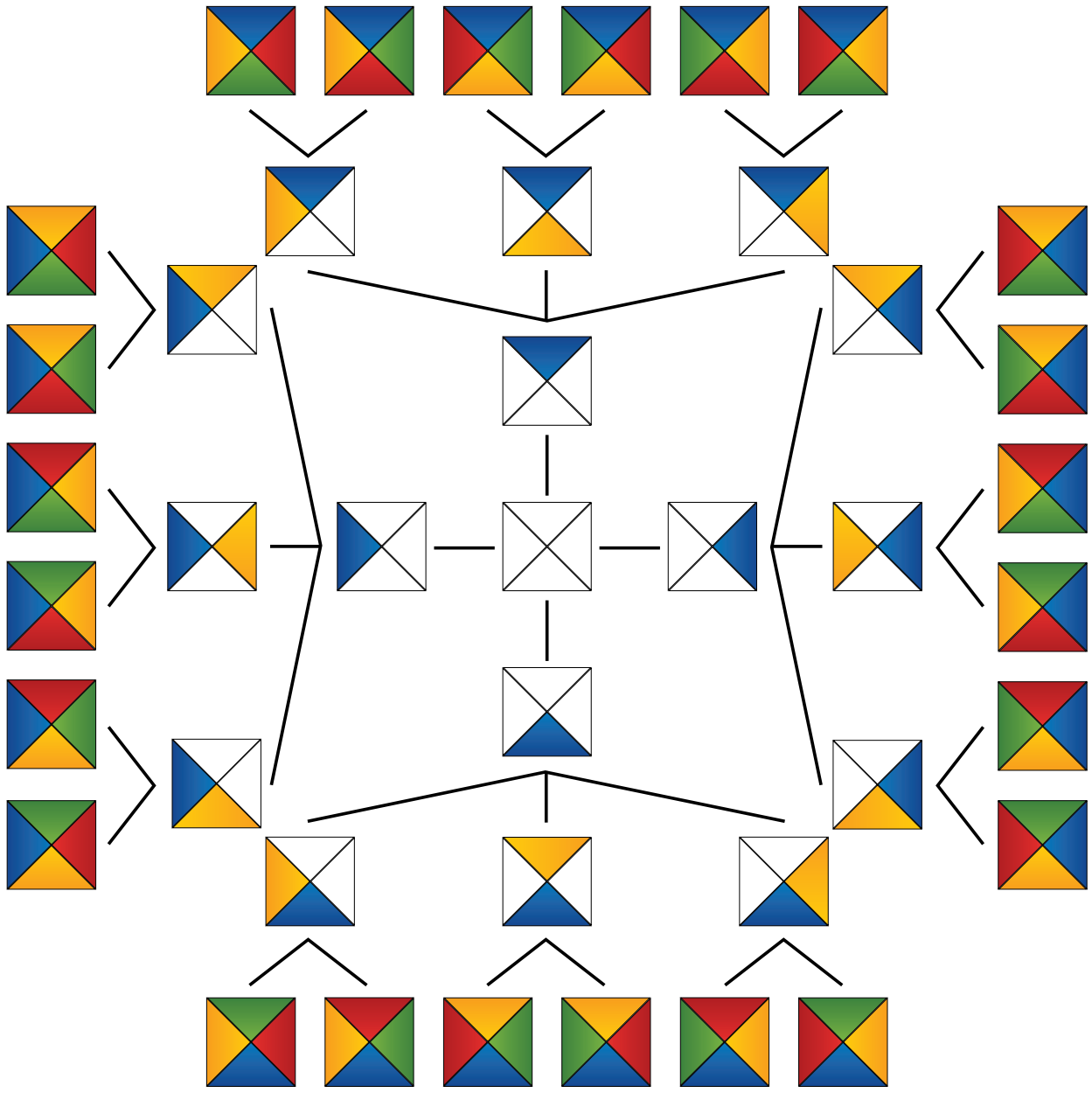


FLOWERINGS



We start with a square and draw its diagonals.
Four isosceles right triangles are thus created, triangles that will be colored with all four colors present and different, blue, yellow, red and green.
Example, we can get the first disposition on the opposites.
Following this rule, we want to know how many different pieces are possible to obtain and not to forget any.
For this, we will logically proceed: attaching a first color, blue, and look for all possible arrangements for this triangle. Then, color the neighboring triangle of another color among the remaining three alternatives and continue so.
What remains now is to remove the identical dispositions so that what remains are the different pieces corresponding to the initial conditions set.

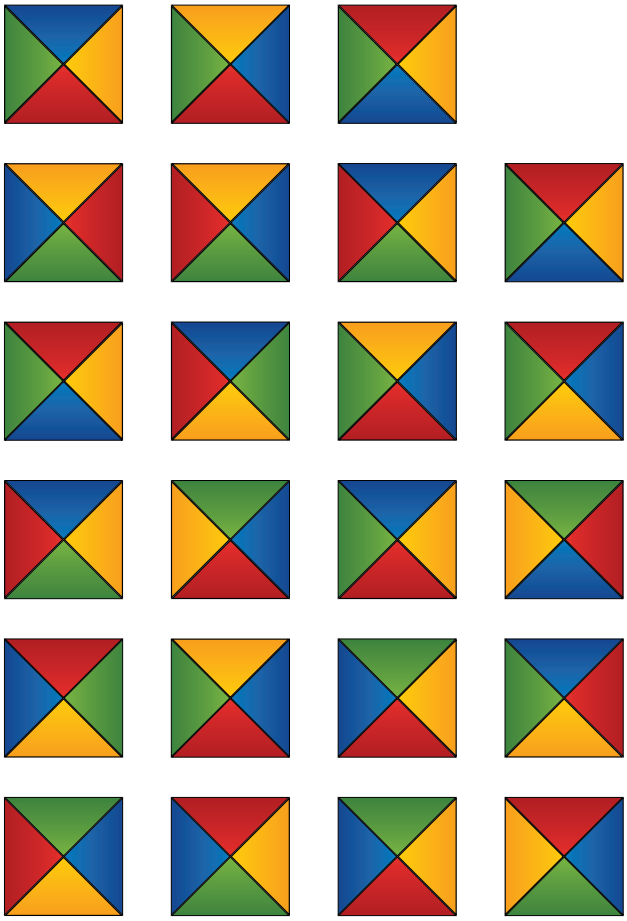
Encircle the identical pieces with the same color.

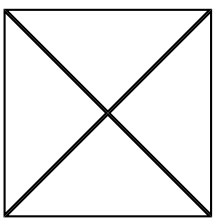


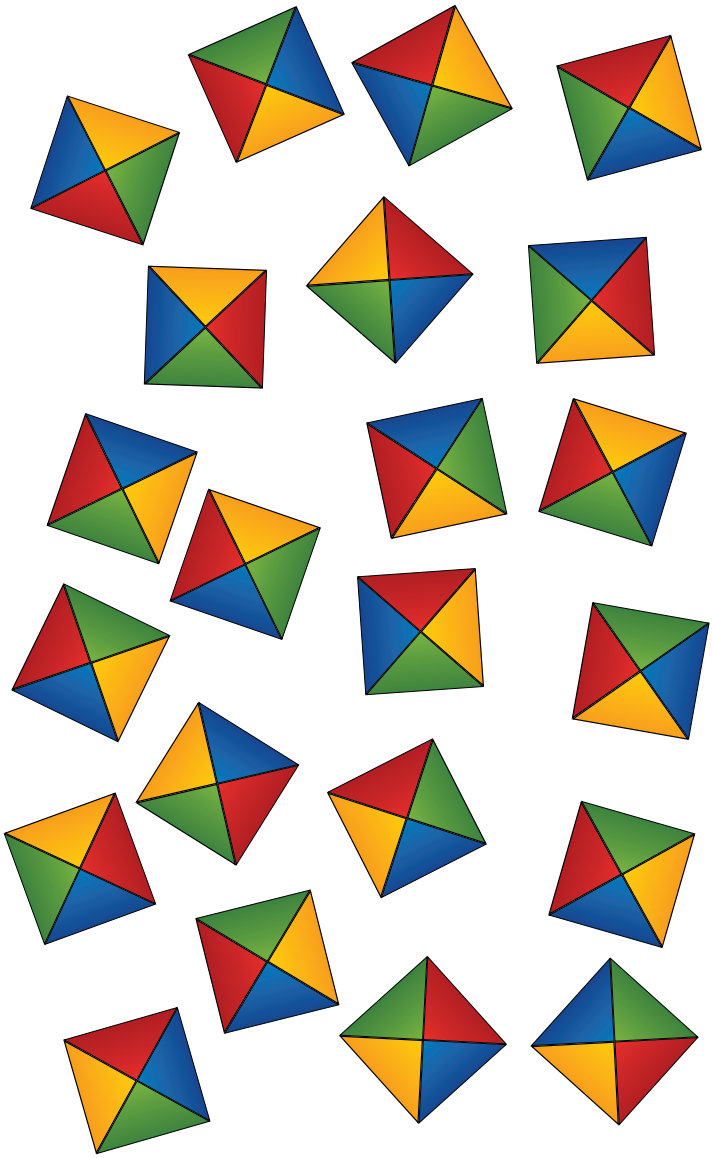
ELIMINATIONS

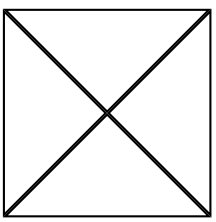
While arranging the 24 pieces of the game, we lost one piece.
Wishing to continue playing, we must identify which colored square was lost.

Can you determine which piece has been lost?
Draw your proposal on the space left open?









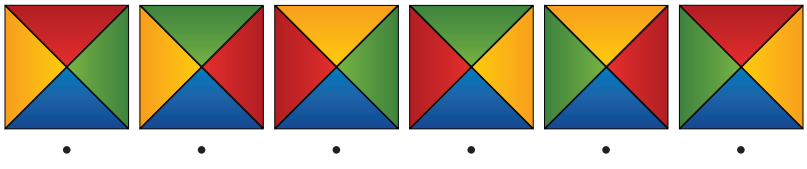
IDENTIFICATIONS

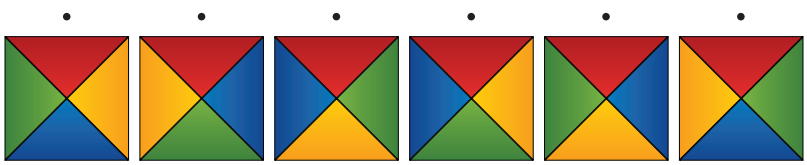
We now have six different square pieces.

Would you be able to recognize them quickly?

A set of six pieces was found by another research team. Some doubts remain on the accuracy of their work.
To address these concerns, connect each piece to its double in the reference set.

Reference set





Doubful set

Other sets were mixed in various arrangements. It is therefore highly possible that these six pieces are not always different.
You must, therefore, identify which is different, that is, the pieces presented in multiple copies.

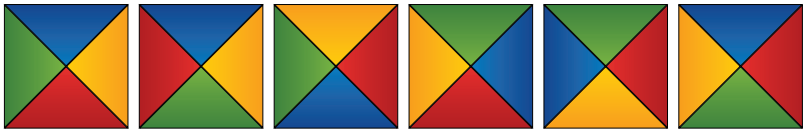
Set 1

A B C D E F

Set 2

A B C D E F

Set 3

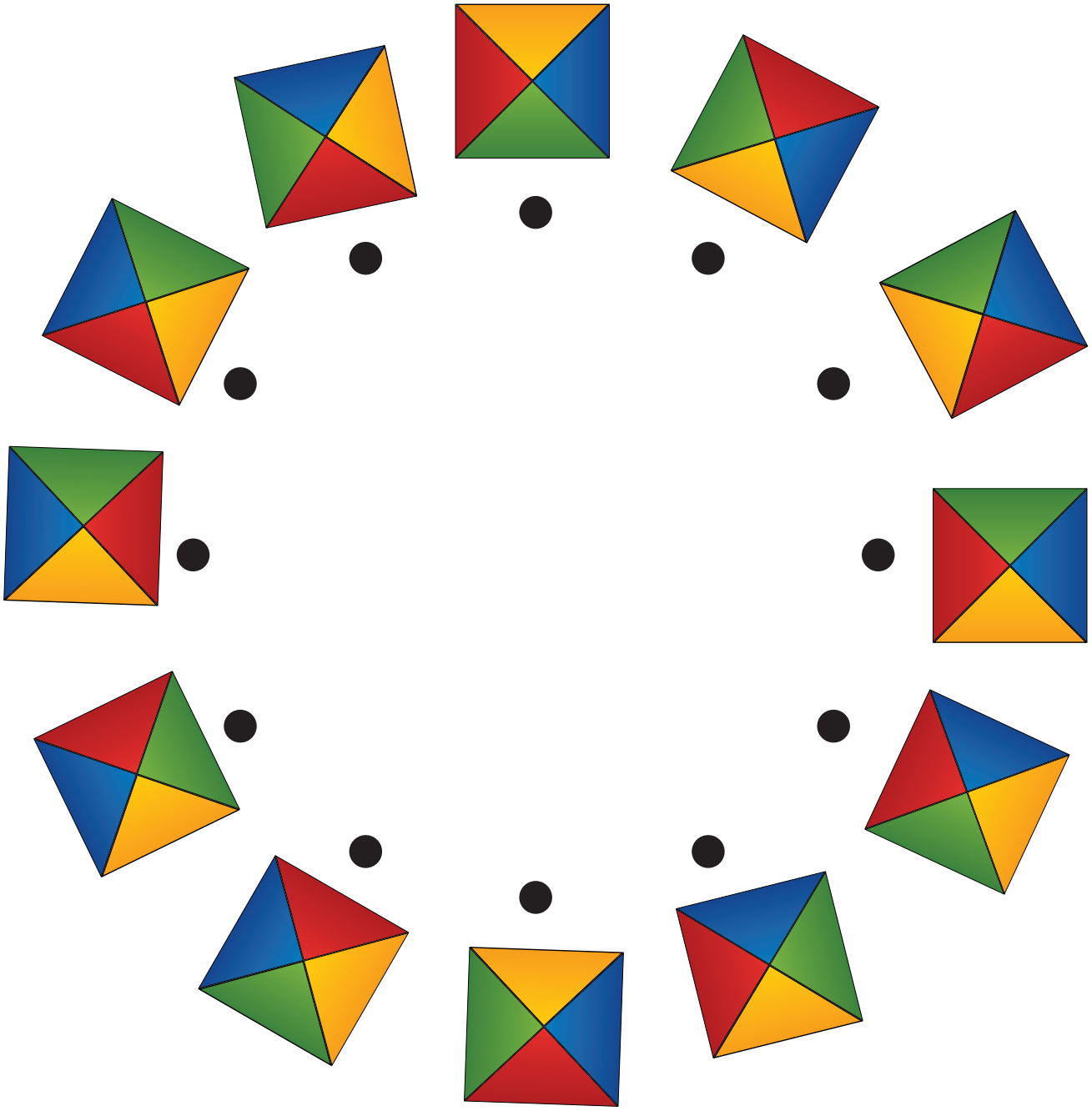


A B C D E F

FARANDOLE

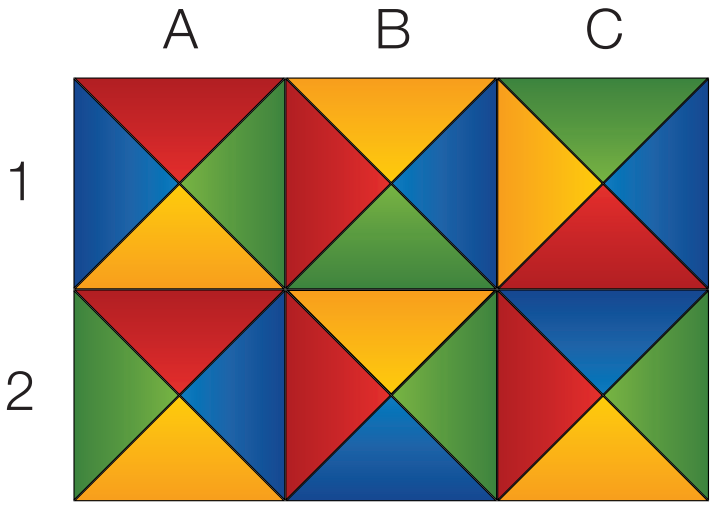
Two sets of pieces were unfortunately mixed.
Knowing that each set contains six different pieces, we would like to isolate these two sets of pieces.

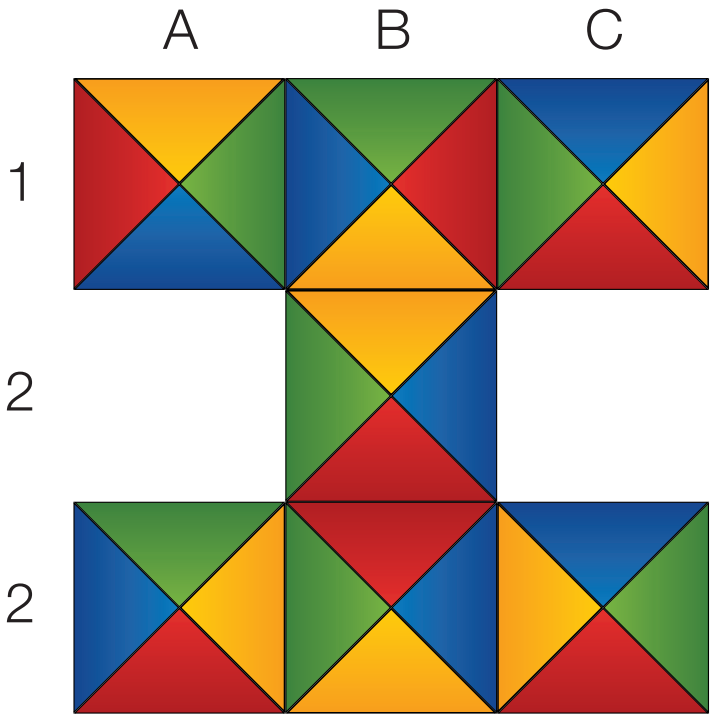
By carefully observing these twelve colored squares, connect the identical pair of squares.



In each of the following constructions, the identical pieces have been used several times.

Identify with the number of the line and column name, the pieces presented in multiple copies.

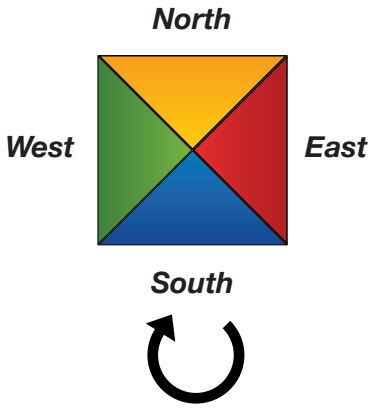




NAMINGS

We now have six square pieces of four colors.

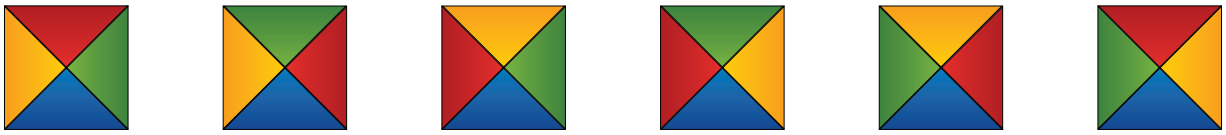
In this set of figures, to identify, to be able to select a piece rather than another, it would be practical to name them. Name them, but how?



Some choices are made once and for all:
· a starting point, the triangle situated in the west
· a course of direction, clockwise
· a designation, the colors in the order they appear on the chosen route.

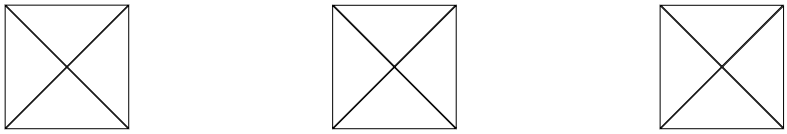
Thus, the opposite piece is named: Green-Yellow-Red-Blue is GYRB.

By following this principle, name the following:



....

You have the names of three parts, try to find them, color them.



BYGR YGBR RYGB

Obvioiusly, the same piece may be rotated and placed in different positions. In this case, just a small problem, its given name changes. One same piece can have several different names.
List all the possible names of these two pieces.

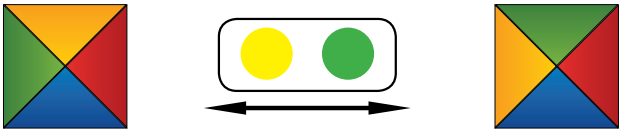
.....
.....
.....
.....

.....
.....
.....
.....

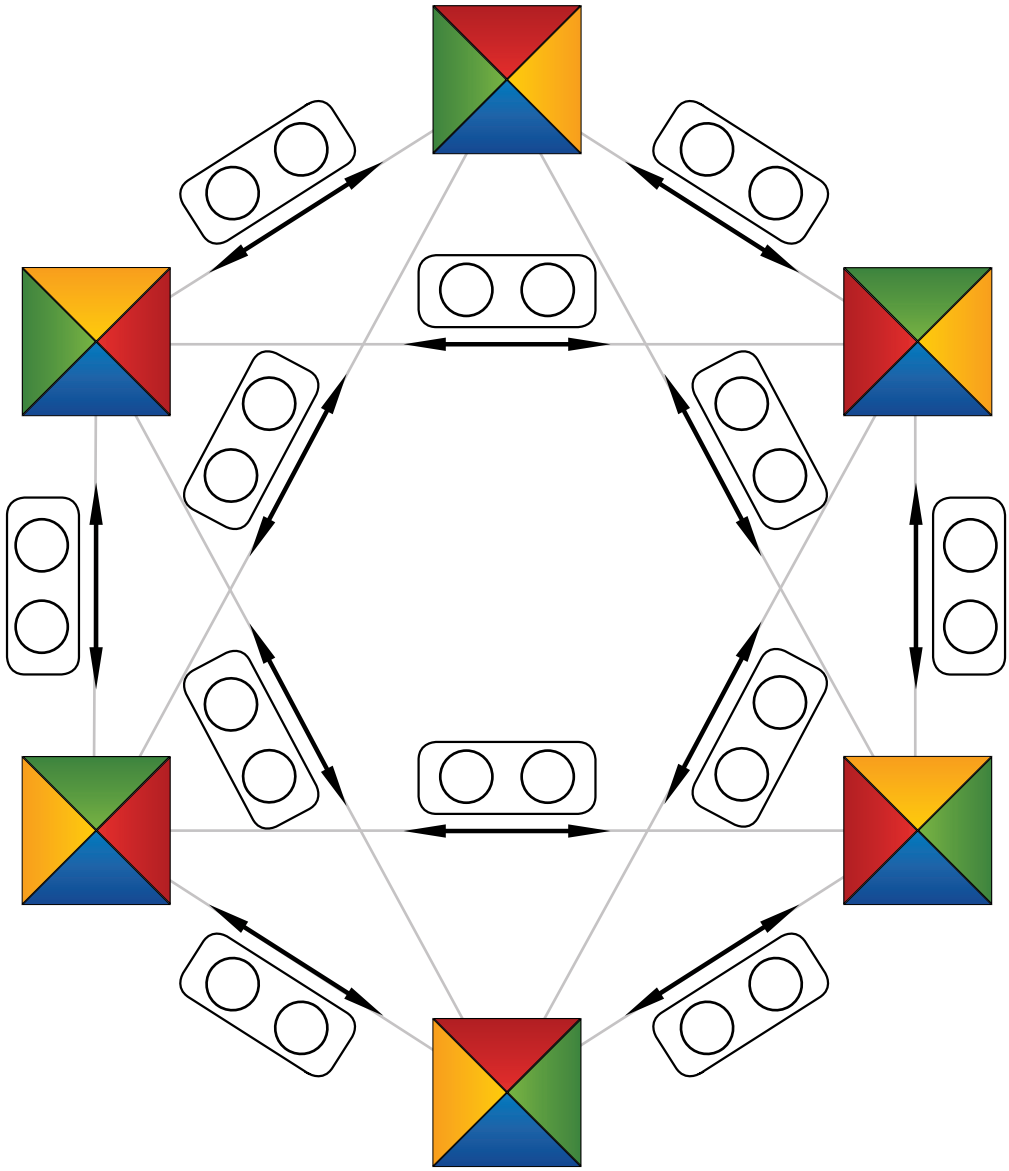
PERMUTATIONS

You now have six square pieces of four colors.

Your goal will be to determine how to move from one to another by only allowing to swap two neighboring colors. For example, you can move the left piece to the right by exchanging the arrangement of the two colors, green and yellow. This possibility will be represented as follows:



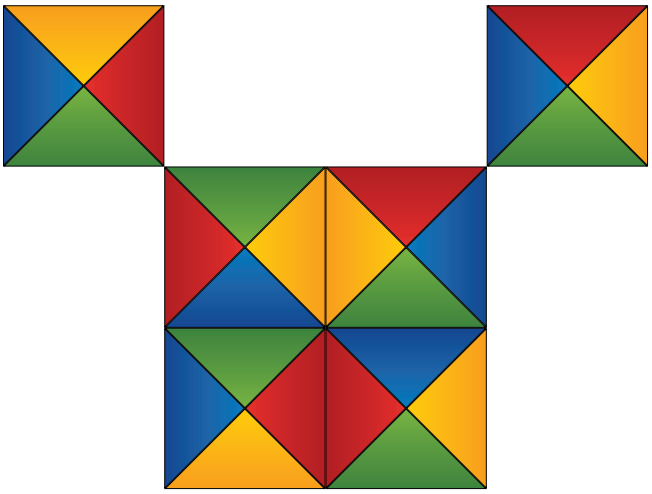
Complete the following diagram by coloring the different pastils which correspond to an exchange of two adjacent colors.

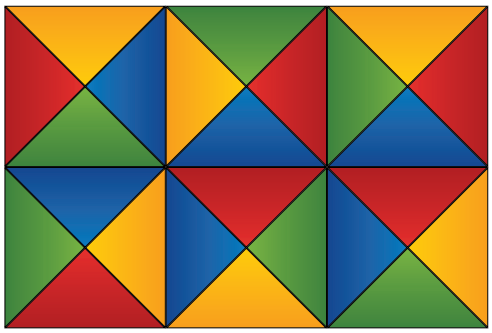


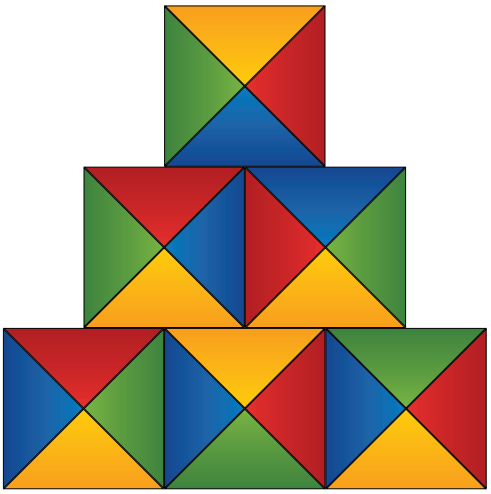
HARLEQUIN 1

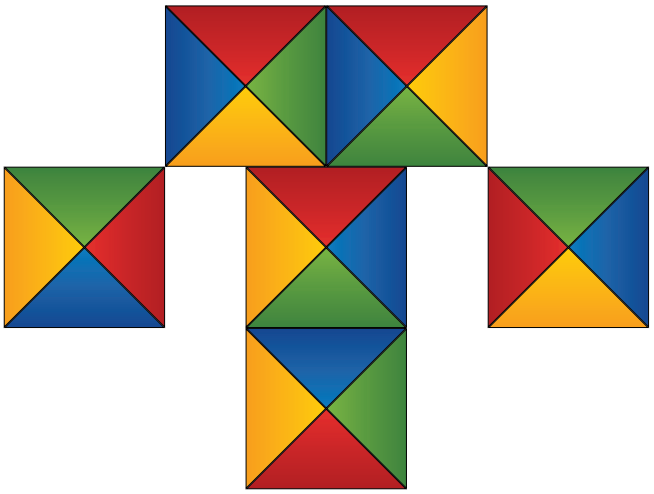
You have a set of six different square pieces and four colors.

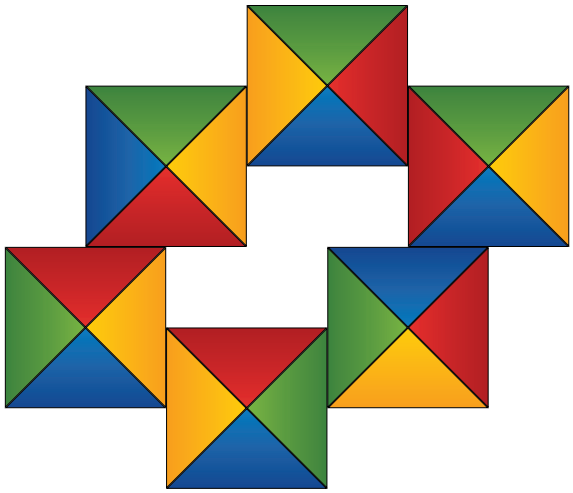
By using them all, reproduce the different models proposed.

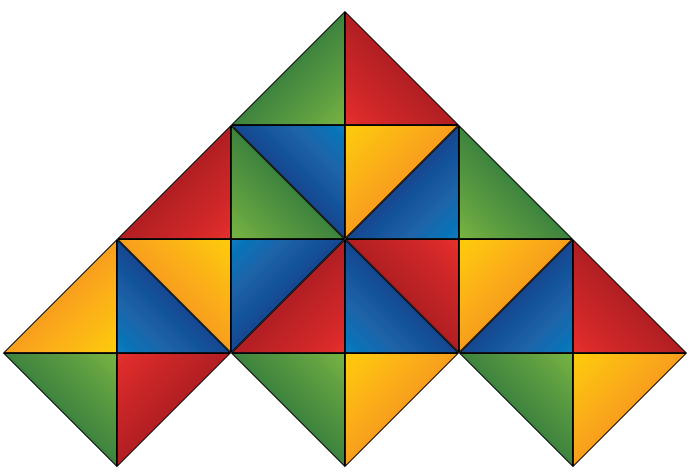








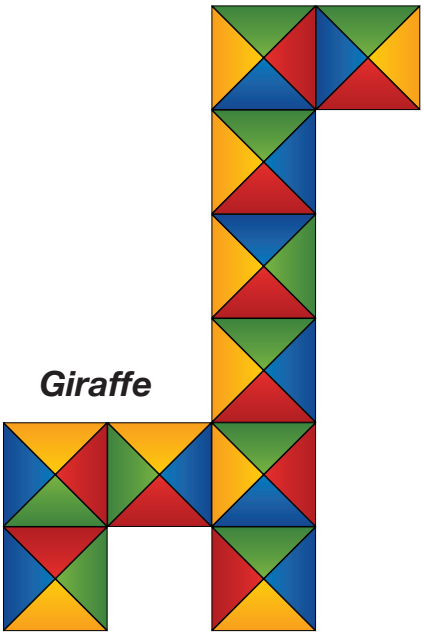




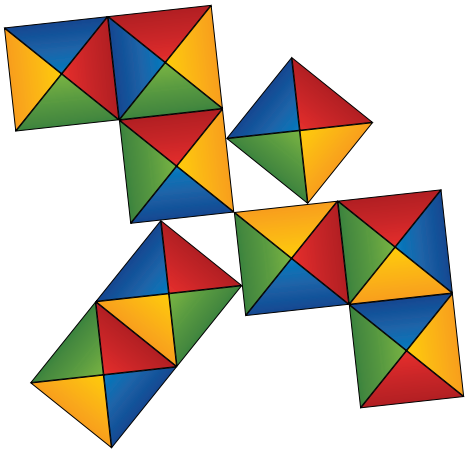
HARLEQUIN 2

You have two different sets of six square pieces and four colors.

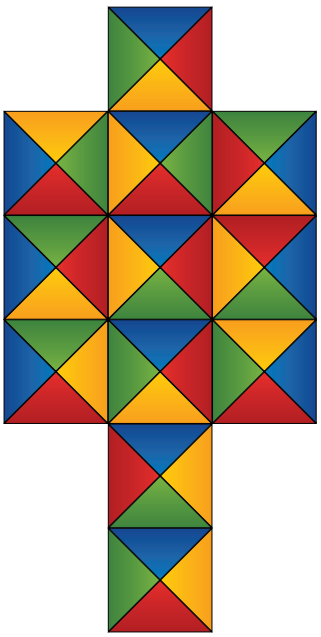
Using some of these twelve pieces, reproduce the proposed different animal silhouettes.



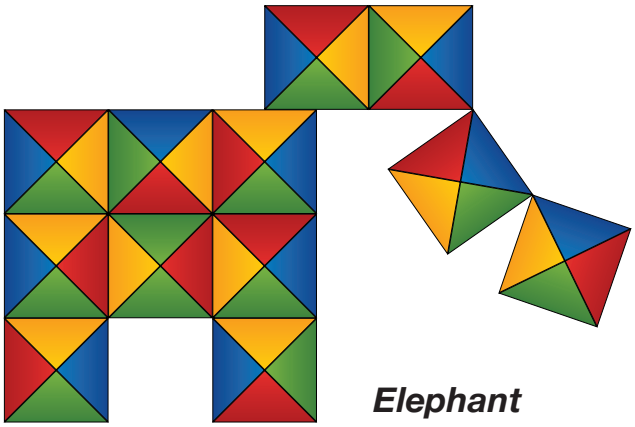
Giraffe



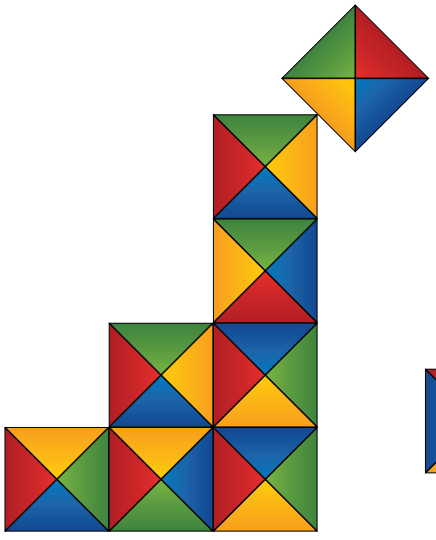
Parrot



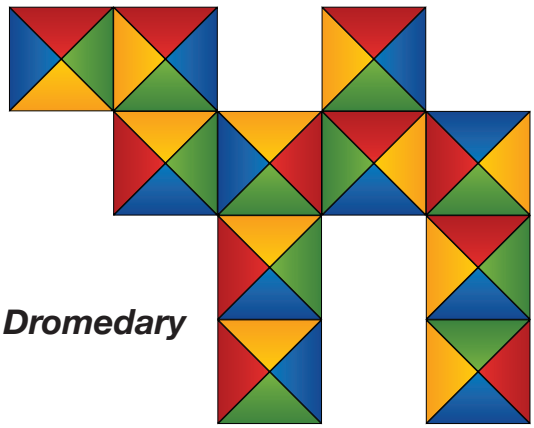
Ray



Elephant

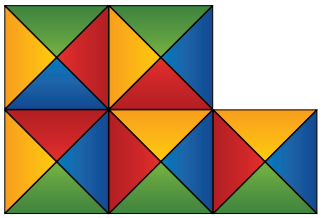


Seal

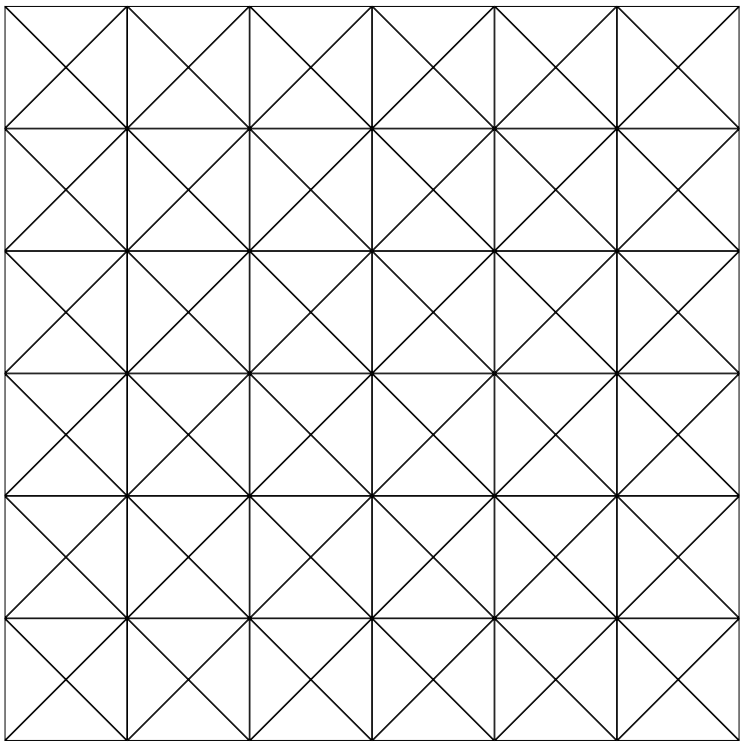


Dromedary

Turtle



HARLEQUIN 3



It's your turn to create some models by initially using all sets of six different pieces and four colors.

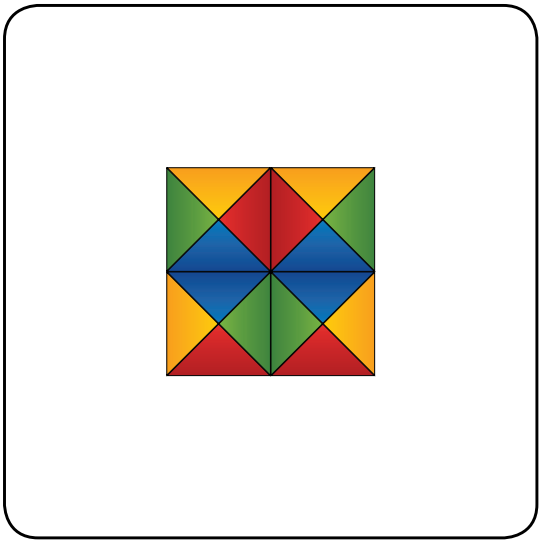
Represent your model in «grid» provided on the opposite side.

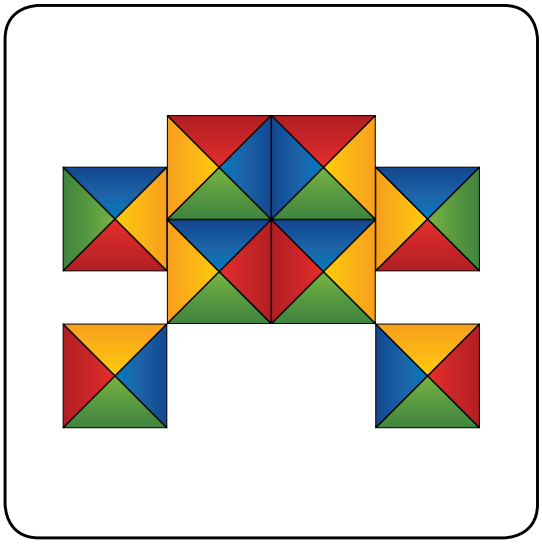
By using the twelve pieces obtained from two complete sets of six different pieces and four colors, create a new «Harlequin». ***Represent your model in the «grid» provided.***

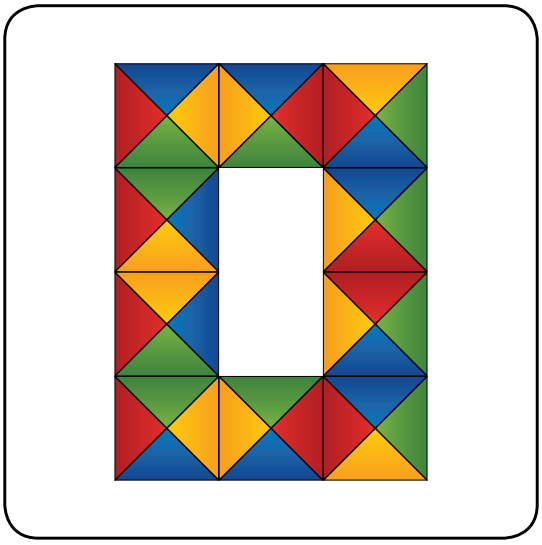
REFLECTIONS

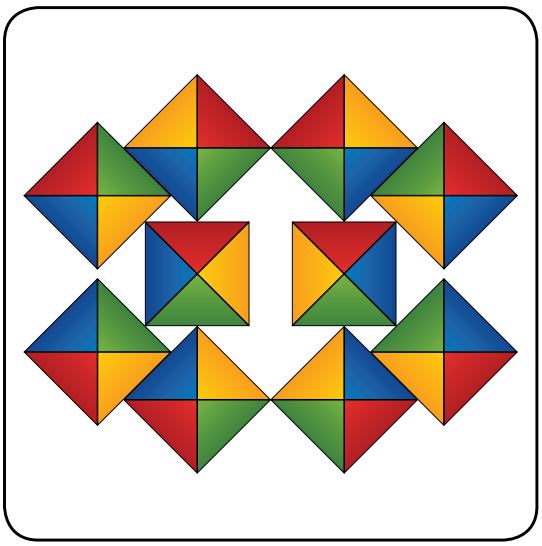
Here you have available a set of 6 different pieces and four colors and a mirror.

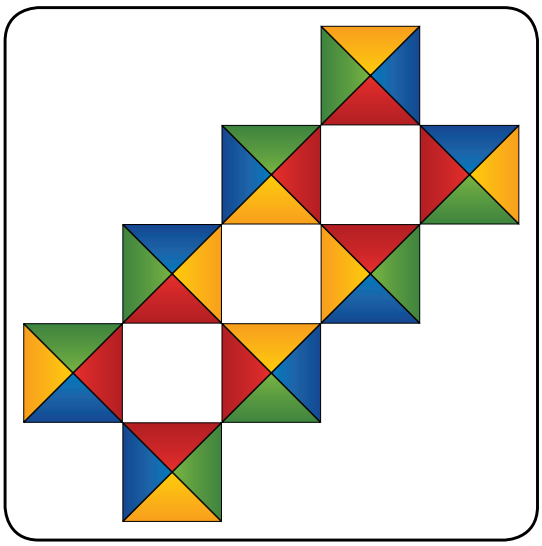
By using some of these four-colored squares and with their reflection in the mirror, you must obtain the following patterns.





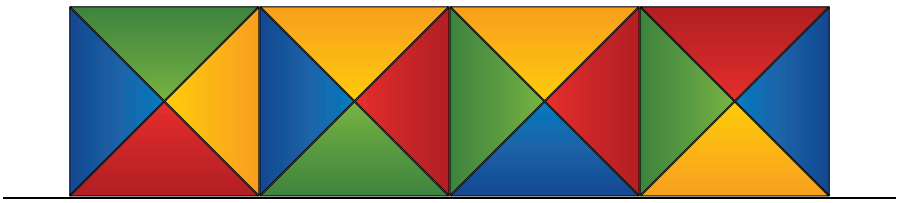


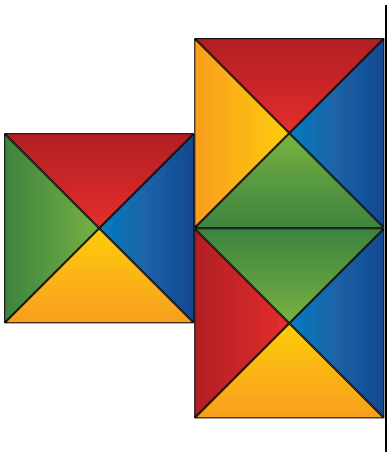


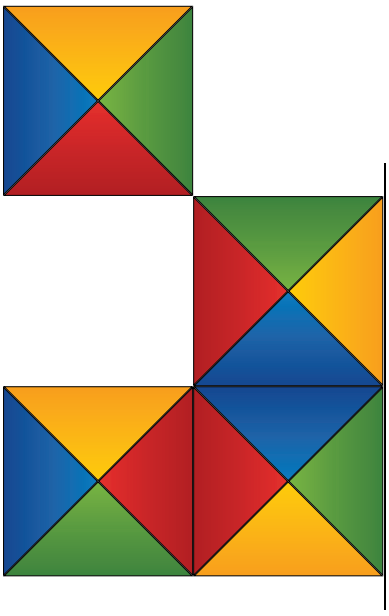


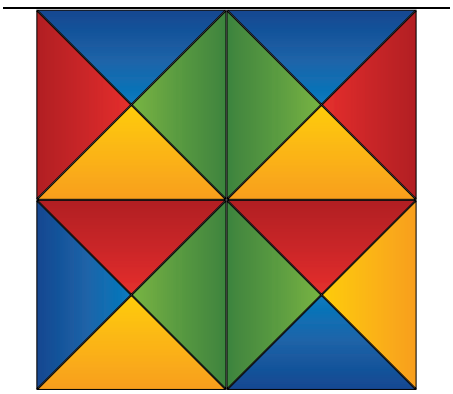
Here you have available a set of 6 different pieces.

Start by reproducing each of these figures by using some of these four-colored squares, you must complete each of them so that in the end, they are symmetrical with respect to the line drawn.



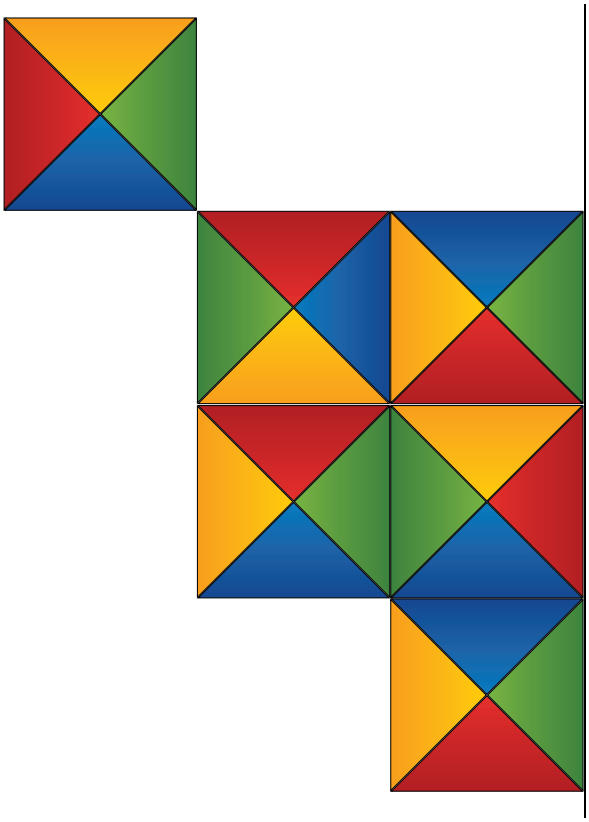


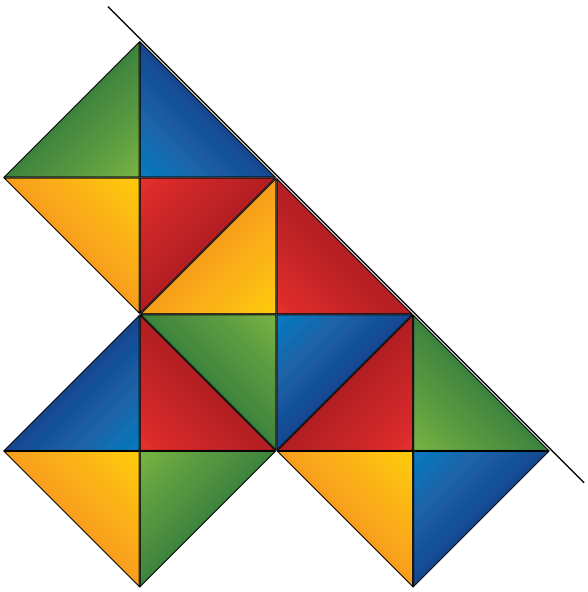


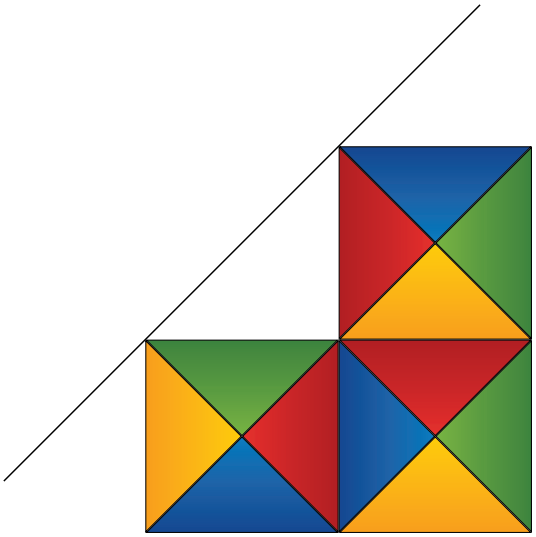


You now have available all the pieces of the game.

By using some of these four-colored squares you must complete each of these figures so that in the end, they are symmetrical with respect to the drawn line.







You now have available all the pieces of the game.

By using some of these four-colored squares you must complete each so that in the end, they are symmetrical with respect to two straight lines drawn.